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Performance of Monthly Multivariate FHS VAR

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PERFORMANCE OF MONTHLY MULTIVARIATE FHS VAR[§]

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Abstract

This study examines the performance of sixteen Value-at-Risk (VaR) models in the context of institutional portfolio management. We focus on multivariate versus univariate approaches of asset modeling, monthly versus shorter risk horizons, and filtered historical simulation (FHS) versus Monte Carlo simulation (MCS) techniques. Tests on VaR violations show that the best performing models are generally the univariate FHS and MCS models with daily asymmetric GARCH specification. A comparative analysis reveals that the asymmetric impact of positive versus negative shocks in the conditional volatility is the most important feature of the models.

JEL *classifications*: G11, G23

Keywords: conditional VaR models, VaR models by filtered historical simulations, GARCH models.

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Abstract

This study examines the performance of sixteen Value-at-Risk (VaR) models in the context of institutional portfolio management. We focus on multivariate versus univariate approaches of asset modeling, monthly versus shorter risk horizons, and filtered historical simulation (FHS) versus Monte Carlo simulation (MCS) techniques. Tests on VaR violations show that the best performing models are generally the univariate FHS and MCS models with daily asymmetric GARCH specification. A comparative analysis reveals that the asymmetric impact of positive versus negative shocks in the conditional volatility is the most important feature of the models.

1. Introduction

Value-at-Risk (hereafter VaR) plays a leading role in evaluating and managing the financial risks taken by institutional investors[1]. For example, VaR measures are useful in disclosing risk to senior executives and institutional shareholders, in monitoring the risk and performance of institutional portfolios such as pension and investment funds, and in managing the risk of market positions of traders and arbitragists. Despite these numerous applications, most studies focus on VaR for calculating the capital requirements of financial institutions regulated by the Basle Committee. These studies reveal that VaR varies widely depending on the methodology and that there is no consensus VaR model adequate in all situations[2]. As a misleading VaR estimate can lead to bad judgment on portfolio risk and, consequently, to bad investment decisions, there is a need for an examination of VaR applications in the context of institutional investors.

In this study, we consider two issues that are relevant for institutional investors looking for a satisfactory VaR. First, as institutional portfolios are selected combinations of assets, the evaluation of portfolio risk could benefit from disaggregating the portfolio returns into its underlying asset returns. It is thus interesting to contrast multivariate VaR approaches, with the modeling of the returns on each portfolio asset, to univariate VaR approaches, where only aggregate portfolio returns are modeled. In particular, it is worth investigating whether multivariate estimation leads to out-of-sample VaR predictions that are more effective as the information pertaining to each asset is taken into account, or less effective as over-parametrization is a possibility.

[1] VaR is defined as a portfolio loss for a given horizon that should only be exceeded at a given target probability.
[2] See Kuester, Mittnik and Paolella (2006)[1] for an overview of this literature and a comparative analysis of a large number of VaR models.

Second, given typical investment horizons of institutional investors, the evaluation of portfolio risk should be made over longer horizon (one month or more) than the VaR horizons prevalent in the literature (one to ten days, following the Basle II agreement). It is not clear that best performing VaR models at very short horizons can continue their success at longer horizons as returns over different horizons do not present the same characteristics. For example, compare to daily returns, monthly returns follow a distribution with less asymmetry and fat tails, are less autocorrelated, and result in a smaller sample of observations for precise estimation. Such characteristics could work against difficult-to-estimate VaR models with complex conditional specification of the return process.

To address these issues, this study investigates the performance of sixteen univariate or multivariate models of monthly portfolio VaR. We consider six conditional filtered historical simulation (hereafter FHS) models (Hull and White, 1998[2]; Barone-Adesi, Bourgoin and Giannopoulos, 1998[3]; Barone-Adesi, Giannopoulos and Vosper, 1999[4], 2002[5]), six conditional Monte Carlo simulation (hereafter MCS) models (Alexander and Leigh, 1997[6]; Ferreira and Lopez, 2005[7]) and four basic unconditional models. The conditional models have time-varying volatilities using either the GARCH specification of Bollerslev (1986)[8], the asymmetric GARCH specification of Glosten, Jagannathan and Runkle (1993)[9] or the RiskMetrics exponential weighting specification of J.P. Morgan & Co[10]. We estimate the models to measure the risk at monthly horizon of an equally-weighted portfolio of three equity indexes (the U.S. S&P 500, the German DAX and the Japanese Nikkei) from 1960 to 2006. Then, we formally assess their out-of-sample predictive ability using the unconditional coverage test, the independence test and the conditional coverage test proposed by Christoffersen (1998)[11].

The VaR approach emphasized in our investigation involves the multivariate FHS technique introduced by Barone-Adesi, Gionnopoulos and Vosper (1999[4], 2002[5]). The FHS technique is a semiparametric method that forecasts the mean and variance of returns through a parametric specification and uses the percentile of the standardized returns in order to calculate the VaR. In a univariate context with a focus on very short horizons, Hull and White (1998)[2], Pritsker (2006)[12], Bao, Lee and Saltoglu (2006)[13], Kuester, Mittnik and Paolella (2006)[1], and Angelidis, Benos and Degiannakis (2007)[14] show that it performs relatively well as it is able to account for the asymmetry, fat tails and changing moments of returns.

Given the issues relevant for institutional investors mentioned earlier, two features of multivariate FHS models are particularly interesting. First, the multivariate simulation involves drawing the standardized error terms of all portfolio assets at a random date to generate simulated asset returns. As discussed by Barone-Adesi, Giannopoulos and Vosper (1999[4], 2002[5]), the grouping of error terms by date preserves the observed co-movements between asset prices, which can be time-varying and more pronounced during extreme events, without requiring difficult-to-estimate conditional correlations. Second, as filtering with a parametric specification can capture the time dependence between returns, it is possible to estimate FHS models using daily data and then compound the simulated daily returns to obtain a VaR at monthly horizon. Chrétien, Coggins and Gallant (2008)[15] show that this strategy improves the performance of FHS VaR at monthly horizon in a univariate setting, as the large number of daily observations allow more precise estimates of the parametric specification. Our selection of sixteen VaR models is in part made to understand the importance of these features.

Our empirical investigation leads to the following findings. In the unconditional coverage test, all models underestimate the risk of the equally-weighted international portfolio at the monthly horizon, as the proportions of VaR violations (i.e. realized losses greater than the VaR) are larger than the 1% or 5% target probabilities. At the 90% confidence level, the tests never reject only four VaR models: the unconditional historical simulation model estimated with monthly returns, the univariate FHS and MCS models with asymmetric GARCH volatility specification estimated with daily returns, and the univariate FHS model estimated with daily GARCH volatility. In the independence test, the results show that unconditional models generate abnormally high proportions of consecutive VaR violations, but support the conditional models. Finally, in the conditional coverage test, the joint examination of the unconditional coverage and independence hypotheses confirms that the best performing models are generally based on an asymmetric GARCH volatility and FHS simulations.

A comparative analysis of the sixteen models highlights the keys to the success of these models. The single most important feature is the asymmetric GARCH specification, which allows negative shocks to have more impact than positive shocks on the conditional volatility. Another feature of the models, namely the FHS technique that accounts for the skewness and fat tails of the realized return distribution, also leads to noticeable improvement in VaR model performance. Finally, the multivariate approach that considers the information in the disaggregate portfolio return is not marginally better performing than the univariate approach, suggesting that the evaluation of monthly portfolio risk does not benefit from the added complexity of the multivariate approach.

The rest of the paper is divided as follow. The next section outlines the sixteen VaR models under consideration as well as some issues regarding their implementation and testing. The third section discusses the data. The fourth section presents and interprets the empirical results. The last section provides our conclusion.

2. Monthly VaR Models

This section describes the monthly VaR models analyzed in this study. The first two subsections detail *univariate* models with MCS and FHS techniques, respectively. These models are univariate in that they fit and simulate directly the aggregate portfolio returns. The third subsection discusses *multivariate* MCS and FHS VaR models that fit and simulate each disaggregate asset returns separately before forming the portfolio. The fourth subsection gives the unconditional VaR models that serve as basic reference to the more sophisticated models. The fifth subsection specifies some implementation choices and introduces the tests of Christoffersen (1998)[11] that assess the performance of the VaR models.

An important aspect of the MCS and FHS models we consider is their use of *daily* data to obtain *monthly* VaR measures. To facilitate the understanding of these models, we can summarize their implementation with a four-step procedure. First, we specify and estimate a return diffusion process for daily returns using data up to the last day of the month preceding the VaR measurement. Second, we simulate the return diffusion process into the future to obtain paths of daily simulated or pseudo returns with a number of observations corresponding to the number of working days in the month of the VaR measurement. Third, we compound the daily simulated returns in each path to find its corresponding monthly simulated return. Fourth, we draw from all the simulation paths the monthly simulated return that corresponds to the target probability of VaR violation. We now turn to a detailed description of these steps for each of the VaR models.

2.1 Univariate MCS VaR Models

The univariate MCS VaR models presume that portfolio returns (or aggregate returns) follow a conditionally Normal distribution. They specify the first moments with a MA(1) model[3] and the second moments with a RiskMetrics, GARCH or asymmetric GARCH models. Specifically, as in Christoffersen (2003), we describe the return diffusion process for day t [R_t] as follows:

$$R_t = c + \phi\ (\sigma_{j,t-1}\eta_{t-1}) + \eta_t \sigma_{j,t}\text{, for } t = 1,\ldots, T, \tag{1}$$

where $\eta_t \sim N(0,1)$. The parameter c represents the mean return, while the parameter ϕ measures the time dependence. The variable $\sigma_{j,t}$ is the conditional standard deviation of the error terms, which is estimated from one of three cases of conditional variance model *j*.

In the first case, $j = RM$, the RiskMetrics exponential weighting model of J.P. Morgan and Co[10]. describes the conditional variance as a function of the past squared return and the past conditional variance:

$$\sigma^2_{RM,t} = (1-\lambda)R^2_{t-1} + \lambda\sigma^2_{RM,t-1}, \tag{2}$$

[3] This choice is based on a comparison of various ARMA specifications using the tests of Ljung and Box (1979)[16]. Results (not reported) show that the MA(1) model proves to be the most powerful specification.

where the parameter λ must be less than one. As this parameter moves away from the unit value, there is greater emphasis on the first past squared return and less emphasis on all the other past squared returns. In this study, we set $\lambda = 0.94$, one of the value suggested by RiskMetrics for evaluating second moments[4].

In the second case, j = *GARCH*, the GARCH(1,1) model specifies the conditional variance as follows (Engle, 1982; Bollerslev, 1986):

$$\sigma^2_{GARCH,t} = \omega + \alpha(\sigma_{p,t-1}\eta_{p,t-1})^2_{GARCH,t-1} + \beta\sigma^2_{GARCH,t-1}, \qquad (3)$$

where the parameter ω is related to the unconditional variance, the parameter α corresponds to the ARCH effect and captures the link with the past squared error term [$(\sigma_{p,t-1}\eta_{p,t-1})^2_{GARCH,t-1}$], and the parameter β represents the GARCH effect, measuring the autocorrelation of the conditional variance (Bollerslev, 1986[8] and Chou, 1988[17]).

In the third case, j = *GJRGARCH*, the GJRGARCH(1,1) model introduces an asymmetrical effect of the positive and negative error terms on the conditional variance following Glosten, Jagannathan and Runkle (1993)[9] and Engle and Ng (1993)[18]. The conditional variance model may be written as follows:

$$\sigma^2_{GJRGARCH,t} = \omega + \alpha(\sigma_{p,t-1}\eta_{p,t-1})^2_{GJRGARCH,t-1} + \beta\sigma^2_{GJRGARCH,t-1} + \gamma I^-_{GJRGARCH,t-1}(\sigma_{p,t-1}\eta_{p,t-1})^2_{GJRGARCH,t-1}, \quad (4)$$

[4] The *RM* specification also assumes a constant conditional mean, so that $\phi = 0$ in equation (1). For more details regarding the model, see Christoffersen (2003)[19], Jorion (2006)[20] or J.P. Morgan & Co.'s[10] related technical documentations.

where the parameter γ measures the asymmetrical effect of negative error terms, since I^- is a binary variable taking the value one if the error term is negative and zero otherwise. The asymmetric effect can be justified by the increased leverage (debt ratio) of companies following a decline in their equity returns, resulting in their increased risk as measured by their variance.

Once Equation (1) has been estimated for each conditional variance model *j* with daily returns up to day *T*, we simulate the return diffusion process into the future to obtain paths of simulated returns. Specifically, we generate random variables [$\breve{\eta}_{ji,T+1}$] from a standardized Normal distribution, *N*(0,1). The simulated return for the first day (at *T* +1) of path *i* then corresponds to:

$$\breve{R}_{ji,T+1} = c + \phi\,(\sigma_{j,T}\eta_{j,T}) + \breve{\eta}_{ji,T+1}\sigma_{j,T+1}, \tag{5}$$

for *j* = *RM*, *GARCH* or *GJRGARCH*. This first simulated return is a function of the last error term [$(\sigma_{j,T}\eta_{j,T})$] from the estimation of Equation (1), and the product of the random variable and the standard deviation estimated from the conditional variance model *j*. Paths of *M* simulated daily returns are similarly obtained by generating as many random variables as there are working days in the month and by reevaluating at each day the conditional means and variances using the different conditional variance models.

We next compound the *M* daily simulated returns to obtain a monthly return [$\breve{R}_{ji,T+1:T+M}$] for each path *i* = 1,…, *MC*, where *MC* is the total number of generated paths. The monthly MCS VaR can finally be measured by drawing from the *MC* paths the monthly pseudo-return that corresponds to the target probability p = pr($R_p < -VaR_p$) of VaR violation:

$$VaR^{p}_{j,T+1:T+M} = -Percentile\left(\left(\breve{R}_{ji,T+1:T+M}\right)_{i=1}^{MC}, 100p\right), \tag{6}$$

for j = *RM*, *GARCH* or *GJRGARCH*. Each month, we evaluate three monthly univariate MCS VaR models corresponding to the three conditional volatility specifications. In the empirical section, we denote these models as UMCSRM, UMCSGARCH and UMCSGJRGARCH, respectively. The next subsection discusses the monthly univariate FHS VaR models.

2.2 Univariate FHS VaR Models

Since financial returns cannot be precisely characterized by a theoretical distribution, several institutions opt for the empirical distribution of realized returns to calculate their VaR measures. By construction, the empirical distribution accounts for the asymmetry and fat tails in returns. But the shortcoming of this approach is that it presumes that returns are independent, which is not supported by the empirical literature (Engle, 1982[21]; Lo and MacKinlay, 1990[22]; etc.). To address the dependence in returns, Barone-Adesi, Bourgoin and Giannopoulos (1998)[3] and Hull and White (1988)[2] propose a VaR with a filtered historical simulation that controls for the autocorrelation in the first two moments of the distribution, while still accounting for its asymmetry and fat tails. In this study, we estimate monthly FHS VaR models using daily data.

The univariate FHS VaR models presume that return diffusion process of daily portfolio returns (or aggregate returns) can be written as follows:

$$R_t = c + \phi\, \varepsilon_{j,t-1} + \varepsilon_{j,t}, \text{ for } t = 1,\dots, T, \quad (7)$$

where $\varepsilon_{j,t} \sim N(0, \sigma^2_{j,t})$. As in Equation (1), $\sigma_{j,t}$ refers to the standard deviation of the error term obtained from the conditional variance model *j*, for *j* = *RM*, *GARCH* or *GJRGARCH*.

Using estimates from Equation (7), we calculate the standardized error term [$\widehat{z}_{j,t}$] for each day *t*:

$$\widehat{z}_{j,t} = \frac{\varepsilon_{j,t}}{\sigma_{j,t}}, \quad (8)$$

for *j* = *RM*, *GARCH* or *GJRGARCH* and *t* = 1,…, *T*. Although the standardized error terms are theoretically independent and equally distributed, no assumption is explicitly made regarding the asymmetry or thickness of their distribution.

Then, rather than generating random variables from a theoretical distribution like in the MCS method, we randomly draw with replacement from the *T* standardized error terms in order to simulate the return diffusion process. Hence, we assume that the empirical distribution of the standardized and randomized component from Equation (7) is representative of its expected distribution. The simulated return for the first day (*T* +1) of path *i* is then computed as:

$$\widehat{R}_{ji,T+1} = c + \phi\, \varepsilon_{j,T} + \widehat{z}_{ji,T+1}\sigma_{j,T+1}, \quad (9)$$

for j = *RM*, *GARCH* or *GJRGARCH*. The first simulated return is a function of the last error term [$(\varepsilon_{j,T})$] from the estimation of Equation (7) and the product of the drawn standardized error term and the standard deviation estimated from the conditional model j. Similarly, paths of M simulated daily returns are generated to represent the M working days of the month, reevaluating at each day the conditional means and variances to take into account the error terms generated from the draw of the standardized error term of the previous day.

We next compound the M daily simulated returns to obtain a monthly return [$\widehat{R}_{ji,T+1:T+M}$] for each path i = 1,…, *HS*, where *HS* is the total number of generated paths. We finally evaluate the monthly FHS VaR by drawing from the *HS* paths the monthly pseudo-return that corresponds to the target probability p of VaR violation:

$$VaR^{p}_{j,T+1:T+M} = -Percentile\left(\left(\widehat{R}_{ji,T+1:T+M}\right)_{i=1}^{HS}, 100p\right), \tag{10}$$

for j = *RM, GARCH or GJRGARCH*. We evaluate three monthly univariate FHS VaR models to match the three conditional variance specifications. We denote these models as UFHSRM, UFHSGARCH and UFHSGJRGARCH. The next subsection addresses the use of MCS and FHS VaR models with disaggregate asset returns.

2.3 Multivariate MCS and FHS VaR Models

We denote the monthly VaR models described previously as univariate since the conditional means and variances are evaluated each month using aggregate portfolio returns. An alternative is to carry out MCS or FHS on each portfolio asset before aggregating the simulated individual asset returns into the simulated portfolio returns with the appropriate portfolio weights. This multivariate approach could lead to more effective VaR performance as the information pertaining to each asset is taken into account in the modeling, but it could also result in less effective out-of-sample predictive ability as over-parametrization is a greater possibility.

To obtain multivariate MCS VaR models, we first estimate Equation (1) for each of the *N* portfolio assets, and then generate *N* random variables for each process. Correlations between the *N* random variables, assumed constant, are taken into account using a Cholesky decomposition of the covariance matrix[5]. We next calculate the daily pseudo-returns of the overall portfolio using the portfolio weights of the *N* portfolio assets. We form similarly *M* daily simulated portfolio returns corresponding to the *M* working days of the month of the VaR measurement, and repeat the process to generate *MC* paths. Once the daily returns are compounded into monthly returns, we compute the monthly VaR by drawing from the *MC* paths the observation corresponding to the target probability of VaR violation.

Extending the FHS VaR models from a univariate context to a multivariate context is done correspondingly (Barone-Adesi, Giannopoulos and Vosper, 1999[4], 2002[5]). We first estimate Equation (7) for each of the *N* portfolio assets. We then standardize each *T*-observations error term series using the relevant conditional standard deviation series. The simulation involves drawing with replacement a random day *t* and using the standardized error terms associated with

[5] Bollerslev (1990)[23] proposes the constant correlation multivariate GARCH model. We avoid more general multivariate GARCH models with time-varying correlations as their estimation suffers from the curse of dimensionality, faces model-selection problems and requires considerable restrictions. See Audrino and Barone-Adesi (2005a[24], 2005b[25]) for a discussion of these issues and an example of a computationally feasible technique if conditional correlations are warranted for VaR.

that day for all portfolio assets to generate their simulated returns. As discussed by Barone-Adesi, Giannopoulos and Vosper (1999[4], 2002[5]), the grouping of error terms by day maintains the observed co-movements between asset prices, which can be more pronounced during extreme events, without requiring difficult-to-estimate conditional correlations. Next, we calculate the daily pseudo-return of the overall portfolio for each date drawn, and similarly determine M daily portfolio returns for the M working days of the month of the VaR evaluation. Once the daily returns are compounded into monthly returns and HS paths are generated, we compute the monthly VaR by drawing from the HS paths the observation corresponding to the target probability of VaR violation.

Given that we consider three conditional variance models, $j = RM$, *GARCH* or *GJRGARCH*, we obtain three multivariate MCS VaR models and three multivariate FHS VaR models. Hereafter, these models are identified as MMCSRM, MMCSGARCH, MMCSGJRGARCH, MFHSRM, MFHSGARCH and MFHSGJRGARCH. The next subsection presents the unconditional models.

2.4 Unconditional VaR Models

In order to compare the conditional VaR models described previously to more basic reference models, we evaluate four monthly unconditional VaR models. The first model, denoted MUNCPAR, is a parametric model assuming a Normal distribution that calculates the VaR analytically with the historical mean and standard deviation of the monthly portfolio returns. The second model, denoted MHS, performs (unfiltered) historical simulation with monthly portfolio returns to generate the HS paths required to evaluate the VaR. The third model, denoted DHS, consists of a historical simulation with daily portfolio returns. The fourth model, denoted

DUNCMCS, is based on a Monte Carlo simulation using the historical mean and standard deviation of daily portfolio returns. In the DHS or DUNCMCS models, the VaR measures are ultimately computed from the *HS* or *MC* paths of monthly pseudo-returns, obtained from the compounding of daily simulated returns. The next subsection details some implementation choices and introduces the statistical tests used to evaluate the performance of the VaR models.

2.5 Implementation Choices and Performance Tests

To sum up, this study examines 16 monthly VaR models: six univariate models with either MCS (UMCSGARCH, UMCSGJRGARCH, UMCSRM) or FHS (UFHSGARCH, UFHSGJRGARCH, UFHSRM), six multivariate models with either MCS (MMCSGARCH, MMCSGJRGARCH, MMCSRM) or FHS (MFHSGARCH, MFHSGJRGARCH, MFHSRM) and four unconditional models (MUNCPAR, MHS, DHS, DUNCMCS).

To implement the models, we make the following three choices. First, in our estimation of the parameters needed for the out-of-sample VaR measure in a given month, we use a moving window of the previous 15 years of historical returns. Second, when simulations are needed, the number of simulated paths is 1,000 (MC = HS = 1,000). Third, we evaluate the VaR at two target probabilities of VaR violation, p = 1% and p = 5%, in order to examine the robustness of the models to two confidence levels.

To test the performance of the models, we apply three “likelihood ratio” tests proposed by Christoffersen (1998)[11]. First, the unconditional coverage test examines if the observed proportion of VaR violations is on average equal the target probability p of VaR violation.

Second, as a VaR violation should not be predictable with the help of available information, the independence test determines if the empirical probabilities of observing a VaR violation when there is or is not a violation in the previous period are on average the same. Third, the conditional coverage test considers whether a VaR model meets the previous two conditions *jointly*. The next section presents the data used in this study.

3. Data

We estimate and test 16 monthly VaR models using both monthly returns and daily returns from an equally-weighted portfolio of three equity indexes: the U.S. S&P 500 index, the German DAX index and the Japanese Nikkei index. The index returns are kept in their local currency, thus assuming implicitly that the portfolio is hedged against exchange rate risk. We consider American business days in our daily series. When the U.S. equity market is closed, non-American index returns are delayed to the following American business day. The multivariate VaR models use disaggregate returns in a system of three equations, as the portfolio is broken down into three international equity indexes. The monthly returns cover the period from January 1960 to November 2006, a total of 563 observations. The daily returns start on January 4, 1960, and end on November 30, 2006, providing a total of 11 750 observations.

Table 1 contains a statistical summary of the daily or monthly domestic index returns. The daily and monthly returns are relatively similar, although the German and Japanese indexes are more volatile. The diversification effect is important since the equally-weighted portfolio of the three indexes shows the lowest standard deviation. The extreme negative returns can be attributed to various financial market shocks, including the crash of 1987.

[Insert Table 1 here]

Table 1 also presents Jarque-Bera test results on the Normality of return distributions, and Ljung-Box test results on the serial correlation of returns (Q-test) and squared returns (Q^2-test) for the past five returns (k=5). For both daily and monthly returns, the Jarque-Bera tests reject the hypothesis of Normality and the Q^2-tests reject the hypothesis of no serial correlation in squared returns. The Q-tests reject the hypothesis of no serial correlation in daily returns (except for the Nikkei index), but not in monthly returns. These results suggest that VaR models that account for serial correlation as well as asymmetry and fat tails in return distribution should perform better.

4. Empirical Results

This section describes our empirical results. We emphasize a comparison of the VaR models by the following features: univariate versus multivariate, FHS versus MCS, GARCH versus GJRGARCH versus RM, and conditional versus unconditional. The first subsection proposes summary statistics on the VaR measures at the 5% and 1% target probabilities. The second subsection deals with the performance of the VaR models, undertaking an analysis of their ability to produce VaR violations with expected frequencies through an examination of tests for unconditional coverage, independence and conditional coverage.

4.1 Descriptive Statistics for Monthly VaR Measures

Table 2 presents descriptive statistics of the monthly VaR measures for an equally-weighted portfolio of the S&P 500, DAX and Nikkei indexes. It shows the mean, standard deviation, minimum and maximum of the VaR measures for the 16 models at the 5% and 1% target probabilities. With 563 monthly return observations and a fifteen-year estimation window, we obtain a total of 383 out-of-sample VaR measures. At the 5% target probability, the VaR measures average between 4.4% (for DHS) and 5.9% (for UFHSGJRGARCH), reaching maxima from 7.5% (for MUNCPAR) to 22.3% (for UFHSGJRGARCH). At the 1% target probability, they average between 6.5% (for DUNCMCS) and 10.7% (for MHS), with maxima from 10.7% (for MUNCPAR) to 44.2% (for UFHSGJRGARCH).

[Insert Table 2 here]

Comparing univariate models, which use aggregate portfolio returns, to multivariate models, which consider portfolio assets individually, otherwise similar VaR models generally have higher mean and volatility in univariate estimation. This finding is especially visible at the 1% target probability and for the GARCH and GJRGARCH volatility specifications. Higher average VaR measures are more prudent assessment of risk, but overstating risk could lead to transaction and opportunity costs for a portfolio with a VaR constraint. The performance tests later explore which type of models is the most accurate representation of risk, thus investigating the issue of whether the modeling of individual assets leads to useful information or over-parametrization in VaR predictions.

Focusing on corresponding FHS versus MCS models, the mean, standard deviation and maximum of VaR measures with FHS are generally higher, especially at the 1% target probability. We attribute this finding to the asymmetry and fat tails in returns that are incorporated in the empirical distribution behind FHS, but not in the Normal distribution underlying MCS.

Concerning the choice of conditional variance models, while there is no clear pattern in standard deviations, the GJRGARCH specification consistently produces higher mean VaR measures than the other two specifications. This result highlights the role of an asymmetrical volatility effect that allows negative shocks to have more impact than positive shocks.

Regarding conditional versus unconditional models, the conditional VaR measures have higher volatility, reflecting their ability to quickly adapt to new financial information. They also show higher mean than the unconditional models using daily data, but similar mean than the ones using monthly data. We explain this difference by the presence of positive autocorrelation in daily returns. It results in realized monthly returns more extreme than their simulated counterparts that are generated assuming independence in daily returns.

4.2 Performance Tests of Monthly VaR Models

4.2.1 Unconditional Coverage Tests

Table 3 (in the first two columns for each target probability) reports the results of the unconditional coverage test, which verifies whether the observed proportion of VaR violations corresponds to what is expected at the 5% or 1% target probabilities. The results show that the realized proportion of VaR violations is higher than the target probability in all cases. For example, the observed proportions vary between 5.7% (for MHS) and 9.1% (for DHS) at the 5% target probability, and between 1.0% (for MHS) and 4.9% (for DUNCMCS) at the 1% target probability. The VaR models studied thus underestimate the risk of the portfolio at the monthly horizon.

[Insert Table 3 here]

The only models not rejected at the 90% confidence level by the unconditional coverage test are the MHS, UFHSGARCH, UFHSGJRGARCH and UMCSGJRGARCH models, which have respectively observed proportions of VaR violations of 5.7%, 6.5%, 6.0% and 6.0% at the 5% target probability and of 1.0%, 1.8%, 1.8% and 1.8% at the 1% target probability. Except for the MHS model, these models are based on univariate simulations with a GARCH-type volatility specification. An analysis of their features reveals why these models do relatively better than the other models.

First, the unconditional coverage results are noticeably better when choosing the GJRGARCH specification rather than the GARCH or RM specifications. In fact, the inclusion of the asymmetrical GARCH volatility effect is the most important feature in monthly VaR modeling since only one of the eight cases involving this specification is rejected at the 95% confidence level. Second, the use of the realized distribution in the FHS technique rather than the Normal distribution in the MCS technique generally leads to slightly better results, except for RM-type models. Third, the estimation using the aggregate portfolio return instead of the individual asset returns improves the performance of the VaR models. Three of the six univariate VaR models are never rejected while all multivariate models are rejected at least once at the confidence level of 90%. Finally, the simple unconditional models using monthly data (MHS and MUNCPAR) present the best results in the unconditional coverage tests at the 5% target probability. However, the independence and conditional coverage tests will be better suited to examine whether the static modeling of the return distribution in unconditional models is appropriate.

4.2.2 Independence Tests

Table 3 (in the last two columns for each target probability) shows the results of the independence test. This test analyzes whether the proportion of VaR violations when there is a violation in the previous month (which is reported in the table) is significantly different from the one when there is not a violation in the previous month. The results indicate that the independence test does not reject any of the conditional VaR models. Their proportions of consecutive VaR violations vary between 6.1% (for UFHSRM) and 11.8% (for MFHSRM) at the 5% target probability, and between 0.0% (for all but MMCSGARCH) and 6.3% (for MMCSGARCH) at the 1% target probability. The selection of univariate versus multivariate models, FHS versus MCS models, or GARCH versus GJRGARCH versus RM models has negligible influence on these test results.

By contrast, for the unconditional VaR models, six of the eight tests reject the independence hypothesis at the 90% confidence level. In particular, the unconditional VaR models are rejected in all cases at the 1% target probability or when they are estimated with monthly data. The proportions of consecutive VaR violations for the unconditional models that did well in the unconditional coverage tests (MHS and MUNCPAR) are greater than 16.7% at the 5% target probability and 22.2% at the 1% target probability. These results confirm that unconditional models do not adjust adequately to periods of greater market volatility, producing measures that are significantly biased downward following violations.

4.2.3 Conditional Coverage Tests

Table 4 presents the results of the conditional coverage test. This test jointly verifies whether consecutive VaR violations are independent and if the observed proportion of violations corresponds to the target probability. The table also gives the average deviation between the realized return and the VaR when all models simultaneously have a VaR violation. This statistic provides an estimate of the loss incurred beyond the value announced by the VaR. Similar to the findings for the unconditional coverage test, the results show rejections of the VaR models for at least one of the two target probabilities in numerous cases. Average losses beyond the VaR in violation situations are between 3.2% (for MMCSRM) and 5.1% (for DUNCMCS) at the 5% target probability and between 4.7% (for UFHSGARCH) and 8.2% (for DUNCMCS) at the 1% target probability.

[Insert table 4 here]

Corroborating previous results, univariate VaR models with GJRGARCH volatility are once again the best performing models according to the conditional coverage test results since they are rejected in only one of the eight cases. More specifically, the UFHSGJRGARCH, UMCSGJRGARCH and MFHSGJRGARCH models are never rejected at the 1% or the 5% target probabilities, a result also found for the UFHSGARCH and MFHSGARCH models. Furthermore, they generally present the smallest average deviations between the realized return and its VaR measure when violations occur, thus creating less surprises. VaR models estimated with GARCH volatility are rejected in four of the eight cases, more specifically when the MCS technique is involved instead of the FHS technique. As all the FHSGJRGARCH VaR models are never rejected at the 90% confidence level, the most crucial features of the best performing models are the consideration of the asymmetric effect that positive and negative shocks have on the conditional volatility and, less significantly, the fat tail distribution captured by the FHS technique. Finally, the estimation of multivariate instead of univariate parameterizations, with six rejected models each, does not make a significant difference.

5. Conclusion

We study the performance of sixteen monthly VaR models for an equally-weighted equity portfolio of the U.S. S&P 500 index, the German DAX index and the Japanese Nikkei index. We evaluate the monthly VaR measures with either Monte Carlo simulation or filtered historical simulation, and by either modeling directly the daily portfolio returns (the univariate approach) or modeling individually the daily returns on the three portfolio assets (the multivariate approach). For each methodology, we consider three conditional variance models: the GARCH(1,1) specification of Bollerslev (1986)[8], the asymmetric GARCH(1,1) specification of Glosten, Jagannathan and Runkle (1993)[9] and the RiskMetrics specification of J.P. Morgan & Co.[10] Finally, we compare our twelve univariate and multivariate conditional VaR models to four unconditional VaR models.

We implement three tests proposed by Christoffersen (1998)[11] to assess the performance of the out-of-sample VaR predictions of the different models. In the unconditional coverage test, the results suggest that all models undervalue the risk of the equity portfolio. At the 90% confidence level, we cannot reject the hypothesis that the observed proportion of VaR violations is equal to target probabilities for four models: the univariate asymmetric and symmetric GARCH VaR models with filtered historical simulation (UFHSGJRGARCH and UFHSGARCH), the univariate asymmetric GARCH VaR models with Monte Carlo simulation (UMCSGJRGARCH) and the (unconditional) historical simulation VaR model. In the independence test, the results support the conditional models, but are not favorable to the unconditional models. As these static models do not adjust quickly to new market information, they generate abnormally high proportions of consecutive VaR violations. Finally, in the conditional coverage test, which jointly examines the null hypotheses of the unconditional coverage and independence tests, the results confirm that the univariate asymmetric and symmetric GARCH VaR models with filtered historical simulation and the univariate asymmetric GARCH VaR models with Monte Carlo simulation are among the best performing models. However, two more GARCH (asymmetric and symmetric) VaR models estimated in a multivariate FHS framework are also not rejected at the 90% confidence level.

Further analysis of the empirical results determines that the key to the relative success of these models compare to the other models is the use of the asymmetric GJRGARCH specification, which allows negative shocks to have more impact than positive shocks on the conditional volatility. Other features, namely the multivariate approach that considers the information in the disaggregate portfolio returns and the FHS technique that accounts for the skewness and fat tails of the realized return distribution, only lead to mitigated improvements in performance.

6. References

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Table 1
Summary Statistics of the Index Returns

This table presents the mean, standard deviation, maximum, minimum and Jarque-Bera Normality test statistics, as well as the test statistics on autocorrelated returns (Q-tests) and squared returns (Q^2-tests) for the past five returns (k=5). The monthly data series on the U.S. S&P500, the German DAX and the Japanese Nikkei equity indexes cover the period from January 1960 to November 2006, a total of 563 observations. The daily data series for the three indexes cover the period from January 4, 1960, to November 30, 2006, for a total of 11 750 observations. The symbols * and ** indicate that the statistics are significant at the confidence levels of 95% and 99%, respectively.

Indices	Mean	Standard Deviation	Max	Min	Jarque-Bera Test	Q-Test (k=5)	Q^2-Test (k=5)
Monthly							
Portfolio	0.618%	3.826%	12.630%	-16.426%	133.469***	4.309	23.389***
S&P 500	0.600%	4.098%	15.104%	-15.759%	25.750***	5.571	30.974***
DAX	0.585%	5.607%	20.038%	-28.202%	189.318***	1.569	22.776***
Nikkei	0.668%	5.309%	19.551%	-20.814%	42.161***	3.579	57.946***
Daily							
Portfolio	0.030%	0.728%	5.642%	-7.623%	17075.110***	343.970***	1877.500***
S&P 500	0.029%	0.899%	5.573%	-7.113%	9723.420***	81.398***	1976.000***
DAX	0.028%	1.171%	9.278%	-12.812%	26613.340***	40.312***	2204.100***
Nikkei	0.032%	1.145%	13.236%	-17.253%	58444.940***	4.625	1175.400***

Table 2
Summary Statistics of the Monthly VaR Measures

This table presents the mean, standard deviation, minimum and maximum of the monthly VaR measures estimated at the 5% and 1% target probabilities for an equally-weighted portfolio of the S&P 500, DAX and Nikkei equity indexes. The VaR series cover the period from January 1975 to November 2006 for 383 VaR observations. See Section 2 for a description of the VaR models. Refer to Table 1 for a description of the data.

	Pr($R_p < -VaR_p$) = 5%				Pr($R_p < -VaR_p$) = 1%			
VaR Models	Mean	Standard Deviation	Min	Max	Mean	Standard Deviation	Min	Max
Unconditional models								
With Monthly Returns								
MHS	5.931%	1.168%	3.908%	8.783%	10.707%	2.583%	7.381%	14.281%
MUNCPAR	5.448%	0.862%	4.119%	7.455%	7.980%	1.192%	6.277%	10.676%
With Daily Returns								
DHS	4.443%	1.137%	2.941%	7.600%	6.785%	1.660%	4.640%	11.298%
DUNCMCS	4.458%	1.151%	2.706%	7.659%	6.536%	1.616%	4.403%	10.931%
Conditional models								
With Daily Returns								
UFHSGARCH	5.628%	2.394%	2.487%	19.866%	9.533%	4.025%	4.390%	31.066%
UFHSGJRGARCH	5.852%	2.571%	2.983%	22.291%	10.513%	4.799%	5.460%	44.160%
UFHSRM	4.807%	2.398%	1.708%	15.744%	7.975%	4.094%	2.720%	30.538%
UMCSGARCH	5.015%	2.040%	2.622%	18.131%	8.260%	3.310%	4.596%	30.595%
UMCSGJRGARCH	5.843%	2.411%	3.155%	18.512%	10.021%	4.156%	4.978%	33.592%
UMCSRM	5.231%	2.491%	1.998%	15.536%	7.748%	3.739%	3.023%	23.162%
MFHSGARCH	5.242%	2.070%	2.512%	20.421%	8.776%	3.582%	4.223%	34.647%
MFHSGJRGARCH	5.232%	2.031%	2.771%	20.532%	9.088%	3.832%	4.471%	38.372%
MFHSRM	4.803%	2.142%	2.182%	15.109%	8.035%	3.915%	3.186%	26.379%
MMCSGARCH	4.789%	2.014%	2.424%	21.379%	7.452%	3.007%	3.576%	29.325%
MMCSGJRGARCH	5.387%	2.090%	2.734%	21.001%	8.662%	3.566%	4.455%	37.737%
MMCSRM	5.723%	2.640%	2.267%	19.014%	8.280%	3.824%	3.325%	24.459%

Table 3

Unconditional Coverage Tests and Independence Tests of the Monthly VaR Models

This table presents the results of the unconditional coverage tests and independence tests proposed by Christoffersen (1998) of the monthly VaR measures estimated at the 5% and 1% target probabilities for an equally-weighted portfolio of the S&P 500, DAX and Nikkei equity indexes. It contains the observed proportion of VaR violation, the likelihood ratio of the unconditional coverage test, the observed proportion of consecutive VaR violation and the likelihood ratio of the independence test. The symbols *, ** and *** indicate that the statistics are significant at the confidence levels of 90%, 95% and 99%, respectively. See Section 2 for a description of the VaR models. Refer to Table 1 for a description of the data.

	$\Pr(R_p < -VaR_p) = 5\%$				$\Pr(R_p < -VaR_p) = 1\%$			
VaR Models	Uncond. Coverage Test (Prop of Violation)	Uncond. Coverage Test (Likelihood Ratio)	Independ. Test (Prop of Consec. Violation)	Independ. Test (Likelihood Ratio)	Uncond. Coverage Test (Prop of Violation)	Uncond. Test (Likelihood Ratio)	Independ. Test (Prop of Consec. Violation)	Independ. Test (Likelihood Ratio)
Unconditional models								
With Monthly Returns								
MHS	5.744%	0.427	18.182%	4.626**	1.044%	0.008	25.000%	4.960**
MUNCPAR	6.266%	1.201	16.667%	3.543*	2.350%	5.109**	22.222%	6.240**
With Daily Returns								
DHS	9.138%	11.214***	11.429%	0.416	4.439%	24.795***	17.647%	4.534**
DUNCMCS	8.616%	8.752***	12.121%	0.685	4.961%	31.135***	15.789%	3.422*
Conditional models								
With Daily Returns								
UFHSGARCH	6.527%	1.723	8.000%	0.222	1.828%	2.129	0.000%	0.298
UFHSGJRGARCH	6.005%	0.768	8.696%	0.400	1.828%	2.129	0.000%	0.298
UFHSRM	8.616%	8.752***	6.061%	0.514	3.394%	13.658***	0.000%	0.985
UMCSGARCH	8.094%	6.555**	9.677%	0.274	2.611%	6.955***	0.000%	0.591
UMCSGJRGARCH	6.005%	0.768	8.696%	0.400	1.828%	2.129	0.000%	0.298
UMCSRM	7.833%	5.560**	6.667%	0.230	3.394%	13.658***	0.000%	0.985
MFHSGARCH	7.311%	3.792*	10.714%	0.607	1.828%	2.129	0.000%	0.298
MFHSGJRGARCH	7.311%	3.792*	10.714%	0.607	1.828%	2.129	0.000%	0.298
MFHSRM	8.877%	9.950***	11.765%	0.535	2.872%	9.007***	0.000%	0.711
MMCSGARCH	8.094%	6.555**	9.677%	0.274	4.178%	21.806***	6.250%	0.241
MMCSGJRGARCH	7.050%	3.021*	11.111%	0.777	2.611%	6.955***	0.000%	0.591
MMCSRM	7.311%	3.792*	7.143%	0.154	2.611%	6.955***	0.000%	0.591

Table 4
Conditional Coverage Tests of the Monthly VaR Models

This table presents the results of the conditional coverage tests proposed by Christoffersen (1998) of the monthly VaR measures estimated at the 5% and 1% target probabilities for an equally-weighted portfolio of the S&P 500, DAX and Nikkei equity indexes. It contains the average deviation between the return and the VaR when all models have a VaR violation, denoted by $(\overline{R_p + VaR_p} | R_p < -VaR_{p,i})$, and the likelihood ratio of the conditional coverage test. The symbols *, ** and *** indicate that the statistics are significant at the confidence levels of 90%, 95% and 99%, respectively. See Section 2 for a description of the VaR models. Refer to Table 1 for a description of the data.

	Pr($R_p < -VaR_p$) = 5%		Pr($R_p < -VaR_p$) = 1%	
VaR Models	$(\overline{R_p + VaR_p} \| R_p < -VaR_{p,i})$	Conditional Coverage Test (Likelihood Ratio)	$(\overline{R_p + VaR_p} \| R_p < -VaR_{p,i})$	Conditional Coverage Test (Likelihood Ratio)
Unconditional models				
With Monthly Returns				
MHS	-3.915%	5.053*	-5.019%	4.967*
MUNCPAR	-4.145%	4.743*	-6.953%	11.349***
With Daily Returns				
DHS	-5.134%	11.629***	-8.000%	29.329***
DUNCMCS	-5.135%	9.436***	-8.244%	34.557***
Conditional models				
With Daily Returns				
UFHSGARCH	-3.817%	1.946	-4.714%	2.428
UFHSGJRGARCH	-3.811%	1.168	-4.858%	3.876
UFHSRM	-4.270%	9.266***	-6.752%	14.643***
UMCSGARCH	-4.499%	6.829**	-6.104%	7.546**
UMCSGJRGARCH	-3.796%	1.168	-4.852%	2.428
UMCSRM	-3.795%	5.790*	-6.688%	14.643***
MFHSGARCH	-3.974%	4.399	-6.926%	2.428
MFHSGJRGARCH	-4.156%	4.399	-6.442%	2.428
MFHSRM	-4.497%	10.486***	-7.215%	9.718***
MMCSGARCH	-4.490%	6.829**	-6.919%	22.047***
MMCSGJRGARCH	-3.892%	3.799	-6.133%	7.546***
MMCSRM	-3.184%	3.945	-5.974%	7.546***

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