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Préambule

La gestion financière responsable vise la maximisation de la richesse relative au risque dans le respect du bien commun des diverses parties prenantes, actuelles et futures, tant de l'entreprise que de l'économie en général. Bien que ce concept ne soit pas en contradiction avec la définition de la théorie financière moderne, les applications qui en découlent exigent un comportement à la fois financièrement et socialement responsable. La gestion responsable des risques financiers, le cadre réglementaire et les mécanismes de saine gouvernance doivent pallier aux lacunes d'un système parfois trop permissif et naïf à l'égard des actions des intervenants de la libre entreprise.

Or, certaines pratiques de l'industrie de la finance et de dirigeants d'entreprises ont été sévèrement critiquées depuis le début des années 2000. De la bulle technologique (2000) jusqu'à la mise en lumière de crimes financiers [Enron (2001) et Worldcom (2002)], en passant par la mauvaise évaluation des titres toxiques lors de la crise des subprimes (2007), la fragilité du secteur financier américain (2008) et le lourd endettement de certains pays souverains, la dernière décennie a été marquée par plusieurs événements qui font ressortir plusieurs éléments inadéquats de la gestion financière. Une gestion de risque plus responsable, une meilleure compréhension des comportements des gestionnaires, des modèles d'évaluation plus performants et complets intégrant des critères extra-financiers, l'établissement d'un cadre réglementaire axé sur la pérennité du bien commun d'une société constituent autant de pistes de solution auxquels doivent s'intéresser tant les académiciens que les professionnels de l'industrie. C'est en mettant à contribution tant le savoir scientifique et pratique que nous pourrons faire passer la finance responsable d'un positionnement en périphérie de la finance fondamentale à une place plus centrale. Le développement des connaissances en finance responsable est au cœur de la mission et des intérêts de recherche des membres du Groupe de Recherche en Finance Appliquée (GReFA) de l'Université de Sherbrooke.

Depuis la dernière crise financière de 2007-2009 (ou 2008-2010 dans Bekaert et Hodrick, 2012), le Comité de Bâle, dont plusieurs organismes de régulation financière sont membres, requiert des institutions financières qu'elles effectuent un monitoring plus serré de leurs risques financiers. De plus, les gestionnaires de fonds de placement tels que les fonds de pension sont également soumis à une gestion plus responsable des fonds qui leur sont confiés. Cette gestion requiert un suivi du risque encouru lors d'investissements ayant pour but de générer un rendement excédentaire afin de satisfaire les besoins de leurs clients. Dans cet article, nous proposons une généralisation de l'approche économétrique GMM proposée dans notre cahier de recherche GReFA précédent (cahier no. 001-16). Cette approche nous permet d'analyser le modèle de Fama et French (2015) dans un cadre dynamique et ainsi obtenir le profil intertemporel de l'alpha de Jensen et celui du beta à travers les phases du cycle économique. De plus, nous étudions également le risque d'illiquidité via le facteur de Pástor-Stambaugh (2003). Nous espérons que cette approche dynamique permettra d'améliorer l'évaluation du risque et de la performance des fonds confiés aux gestionnaires de portefeuilles.

Rolling Regression Analysis of the Pástor-Stambaugh Model: Evidence from Robust Instrumental Variables

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Abstract

The CAPM, Fama-French (FF), and Pástor-Stambaugh (PS) factor models are examined using a new dynamic rolling regression version of the GMM method. This rolling regression framework not only allows us to investigate phases of the business cycle, but also permits regression estimates to vary through time due to changes in the development and efficiency of the sectors. The principal reasons for using the dynamic GMM with robust instruments is that some of these factors are measured with errors and the disturbances may be non-spherical. The CAPM appears as the most parsimonious model to explain the FF sector returns. Furthermore, the rolling GMM approach is clearly more sensitive to dynamic financial episodes than the OLS approach. In particular, liquidity has some anticipatory power, as it is able to forecast the 2007-2009 crises with heightened volatility starting in late 2005.

Keywords Business cycles; CAPM; Fama-French model; liquidity; rolling GMM; robust instruments

JEL Classification C10; G12

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Introduction

Recently, Racicot and Rentz (2015) investigated in a static context the use of GMM with robust instruments to test whether liquidity has an effect on the Fama-French 12 sector returns. The main objective of this study is to recast the previous study into a dynamic rolling regression framework to determine the role of liquidity during the phases of the business cycle[1].

Factor models, such as the CAPM, Fama-French (FF), and Pástor-Stambaugh (PS) models, are examined using a new dynamic version of the generalized method of moments (GMM) based on robust instrumental variables with rolling regressions. The reason for using a dynamic GMM method is that some of these factors may contain measurement errors. These GMM rolling estimates are compared with ordinary least squares (OLS) rolling estimates to illustrate the difference in results.

The generalized method of moments (GMM) has mostly been applied in static or panel data frameworks in financial econometrics. For example, see Campbell, Lo, and MacKinlay (1997) and Cochrane (2005) who apply GMM to time-series, cross-sectional, and panel data financial regression models. Following the robust instruments approach that recently appeared in Racicot and Théoret (2009), Racicot (2015), and Racicot and Rentz (2015), we extend their static GMM method to a dynamic setting which not only allows us to take into account the phases of the business cycle but also permits regression estimates to vary through time due to changes in the development and efficiency of the sectors. We therefore generalize to our dynamic context their robust instruments that are based on higher moments of the variables.

Extending GMM to a dynamic setting allows us to capture dramatic financial events such as the subprime mortgage crisis of 2007-2009 from which the world economies have not yet fully recovered to this day. Note one finding of this paper is that our rolling GMM approach is clearly more sensitive to dynamic financial episodes than the benchmark OLS method used by practitioners. This might be

[1] Note that Ghysels (1998) found that using a dynamic approach to estimating beta (e.g. a conditional CAPM) may not be appropriate. However, we decided to use a rolling regression methodology to show a novel application of our GMM approach in a dynamic framework.

explained by the fact that the robust instruments used in this study are built on higher moments and cumulants of the observed data. Extreme events should be better represented by higher moments like skewness and kurtosis. The data used in this study are highly non-normal. The Jarque-Bera (1980) statistics for all variables in this study are significant at the 1% level, indicating just such non-normality. Our robust instruments can be viewed as optimally combining cross-skewness and cross-kurtosis which helps the GMM_d estimation process capture the fat-tail events observed in the data.

In addition to the GMM and OLS estimation procedures, we have conducted some multivariate GARCH experiments using the BEKK model of Engel and Kroner (1995). These experiments are used to help in identifying if the illiquidity factor LIQ developed by Pástor-Stambaugh (2003) is dynamically related to the 12 Fama-French (FF) excess sector returns (i.e. risk premiums). In line with our rolling regression experiment, we do not find much explanatory power for the LIQ factor in the data sample. Moreover, we find more dynamic correlation between the FF factor SMB and LIQ than we do between the excess 12-sector returns and LIQ. The SMB factor represents the difference in returns between a portfolio of small cap stocks and big cap stocks. Since the small cap stocks tend to be less liquid than the big cap stocks, it appears the SMB variable already captures most illiquidity effects, rendering the LIQ variable superfluous.

Nevertheless, the LIQ variable may have some usefulness. In particular, the rolling market beta using GMM in the Pástor-Stambaugh (PS) model shows some forward-looking ability to anticipate extreme financial events. The market beta becomes quite volatile beginning in the fall of 2005, first spiking sharply upwards and then even more sharply downwards before the subprime mortgage crisis of 2007-2009 unfolded. The GMM estimation process reveals this asymmetric behavior; whereas, the OLS estimator is essentially fairly smooth. This asymmetric behavior seems with consistent the Black (1976) leverage effect modeled later by Nelson (1991) using an EGARCH approach. The use of the GMM approach is justified by the fact that the PS illiquidity factor LIQ is a constructed variable using a regression approach. Using the constructed variable LIQ as an independent variable in a subsequent OLS regression leads to biased inferences based on standard t tests as Pagan (1984, 1986) and Pagan and Ullah (1988) have shown. Thus, the GMM approach is helpful in avoiding these biased inferences, as the t statistics using GMM are more efficiently estimated.

The remainder of this paper is organized as follows. First, the GMM and OLS rolling regression estimation procedures for the market, 3-factor Fama-French (1992, 1993), and Pástor-Stambaugh (2003) models are presented. Then, tabular results are shown and analyzed for descriptive statistics for the data sample. In particular, we show that the Jarque-Bera statistics for all variables in the PS model are significant at the 1% level. This is strong evidence of non-normality of all of the data. Next, empirical results are analyzed. It appears that the excess return on the market is the only meaningful factor in explaining excess sector returns. Nevertheless, the rolling regression beta for the excess market return seems to anticipate the 2007-2009 financial crisis in the PS model. Finally, we present our conclusions and discuss the limitations of this study.

OLS and GMM$_d$ Rolling Regression Estimation Procedures

OLS Rolling Regression

In the following, we generalize the static regression framework of Racicot and Rentz (2015) to a dynamic one in order to account for the financial fluctuations observed in the sectors of the FF database.

Equation (1) is the well-known market model of Markowitz (1959) and Sharpe (1964) written into the dynamic rolling regression form,

$$Y_t = \alpha_t + \beta_t X_t + \varepsilon_t \tag{1}$$

where Y_t is the excess return of an asset (i.e. asset return minus risk-free return) at time t, α_t is the dynamic rolling Jensen (1968) alpha performance measure, β_t is the dynamic rolling measure of relative systematic risk, X_t is the excess market return (i.e. market return minus risk-free return) and ε_t is a random error term. Note that a matrix of variables at time t includes observations from 60 time periods in our rolling regression. We can rewrite (1) as

$$r_{it} - r_{ft} = \alpha_{it} + \beta_{it}\left(r_{mt} - r_{ft}\right) + \varepsilon_{it} \quad (2)$$

where r_{it} is the return of the ith Fama-French sector, $i = 1, \ldots, 12$, in period t, r_{ft} is the risk-free return in period t, α_{it} is the ith sector 60-month[2] rolling Jensen alpha performance measure for period t, β_{it} is the ith sector 60-month rolling measure of relative systematic risk for period t, r_{mt} is the market return for period t, and ε_{it} is a random error term for the ith sector in period t.

Fama and French (1992, 1993)[3] extended the market model to include two additional factors, the small size anomaly (SMB) and the value anomaly (HML). The dynamic rolling regression generalization of this FF 3-factor model is given by

$$r_{it} - r_{ft} = \alpha_{it} + \beta_{1it}\left(r_{mt} - r_{ft}\right) + \beta_{2it}SMB_t + \beta_{3it}HML_t + \varepsilon_{it} \quad (3)$$

SMB_t is the difference in returns on FF's small and big (i.e. large) cap portfolios. HML_t is the difference in returns on FF's high-book-to-market and low-book-to-market portfolios.

Pástor and Stambaugh (2003) examined a 5-factor model. Their PS model included the 3 factors of FF, the momentum anomaly (MOM) of Carhart (1997), and their liquidity factor (LIQ). The dynamic rolling regression generalization of the PS model is given by

$$r_{it} - r_{ft} = \alpha_{it} + \beta_{1it}\left(r_{mt} - r_{ft}\right) + \beta_{2it}SMB_t + \beta_{3it}HML_t + \beta_{4it}MOM_t + \beta_{5it}LIQ_t + \varepsilon_{it} \quad (4)$$

MOM_t, also known as UMD_t, is the difference in returns of a portfolio of the top 30% of stocks on the NYSE, AMEX, and NASDAQ over the last 11 months with a one-month lag and of a portfolio of the bottom 30% of stocks for the same indices and 11 months. These portfolios are reconstructed for each

[2] Our selection of 60 months follows the convention of Reilly and Brown (2009, p. 219). They note that there is no theoretically correct time interval for estimating returns. They conclude that the 60-month period is widely used by Morningstar and others, for example, and seems to be neither too long nor too short.

[3] Fama and French (2015) have recently introduced two new factors to their original three-factor model. These factors are *RMW* which is a portfolio that is long firms with robust profitability and short firms with weak profitability, and *CMA*, which is a portfolio of firms with conservative investment policy minus firms with an aggressive investment policy. The new five-factor model has the virtue of having some theoretical underpinnings compared to their previous model.

succeeding time period. The LIQ_t factor is the PS liquidity factor and is a constructed variable. LIQ_t is an average of the stock $\hat{\gamma}_{it}$ obtained from regression (5).

$$r_{id+1t} - r_{md+1t} = \theta_{it} + \varphi_{it} r_{idt} + \gamma_{it} sign\left(r_{idt} - r_{mdt}\right) v_{idt} + \varepsilon_{id+1t} \tag{5}$$

where r_{idt} is the return of stock *i* on day *d* in month *t* and v_{idt} is the dollar trading volume of stock *i* on day *d* in month *t*[4].

The Dynamic GMM_d

To account for the phases of the business cycle, we rely on a new dynamic version of the approach previously applied by Racicot and Théoret (2009), and Racicot and Rentz (2015). We therefore generalize their approach as follows.

The GMM_d approach is based on the generalized method of moments developed by Hansen (1982). GMM_d uses robust instruments based on higher moments and cumulants of the sample of explanatory variables, which are called the *d* instruments as these instruments can be viewed as a distance measure. The dynamic *d* instruments can be obtained as follows:

$$d_t = X_t - \hat{X}_t = X_t - P_{zt} X_t = \left(I - P_{zt}\right) X_t \tag{6}$$

In (6) the matrix d_t is a rolling matrix of instruments at time t for 60 observations that can be defined in deviation form as

$$d_t = x_t - \hat{x}_t \tag{7}$$

where x_t and $\hat{x}_t$ are the matrix X_t and $\hat{X}_t$ taken in deviation from their means.

[4] For more detail, see Pástor-Stambaugh (2003). For a discussion of liquidity measures see Johann and Theissen (2013).

Intuitively, the variable d_t is a filtered version of the endogenous variables. It potentially removes non-linearities that might be hidden in x_t. The $\hat{x}_t$ in (7) are obtained applying OLS on the z_t instruments.

$$\hat{x}_t = \hat{\gamma}_{0t} + z_t\hat{\phi}_t \tag{8}$$

The z_t instruments are defined as $z_t = \{z_{0t}, z_{1t}, z_{2t}\}$, where $z_{0t} = i_{Tt}$, $z_{1t} = x_t \bullet x_t$, and $z_{2t} = x_t \bullet x_t \bullet x_t - 3x_t\left[D\left(x_t{}'x_t / T\right)\right]$. The symbol $\bullet$ is the Hadamard product, $D\left(x_t{}'x_t / T\right) = \underset{T\to\infty}{p\lim}\left(x_t{}'x_t / T\right) \bullet I_n$ is a diagonal matrix, and I_n is an identity matrix of dimension n $\times$n, where n is the number of independent variables. z_1 contains the instruments used in the Durbin (1954) estimator, and z_2 contains the cumulant instruments used by Pal (1980). These instruments are the dynamic generalization of Racicot and Théoret (2009) and are closely related to the static instruments of Dagenais and Dagenais (1994).

It should be emphasized that the 3rd and 4th cross sample moments are used as instruments to estimate the model parameters. This is in line with the work of Mandelbrot (1963, 1972) and Fama (1963, 1965) who found that stock returns are not normally distributed. We believe that the assumption of normality is a sufficient condition for the estimators to be consistent once measurement errors are purged using these 3rd and 4th cross sample moments.

The dynamic GMM_d generalization of the robust instrumental variable estimator is as follows:

$$\underset{\hat{\beta}_t}{\arg\min}\left\{n^{-1}\left[d_t{}'\left(Y_t - X_t\hat{\beta}_t\right)\right]' W_t n^{-1}\left[d_t{}'\left(Y_t - X_t\hat{\beta}_t\right)\right]\right\} \tag{9}$$

The variables in (9) are defined below in (10) through (12). We start with W_t, which is a weighting matrix that can be estimated using the HAC[5] estimator and Y_t is defined as

$$Y_t = X_t \beta_t + \varepsilon_t \tag{10}$$

where X_t is assumed to be an unobserved matrix of explanatory variables at time t. Note that a matrix of variables at time t includes observations from 60 time periods in our rolling regression. The observed matrix of observed variables is assumed to be measured with normally distributed error[6], *viz*., $X_t{*} = X_t + v_t$. $\hat{\beta}_t$ is defined as

$$\hat{\beta}_t = \hat{\beta}_{t,TSLS} = \left(X_t{}' P_{zt} X_t \right)^{-1} X_t{}' P_{zt} Y_t \tag{11}$$

P_{zt} is defined as the standard "predicted value maker" or "projection matrix" used to compute

$$P_{zt} X_t = Z_t \left(Z_t{}' Z_t \right)^{-1} Z_t{}' X_t = Z_t \hat{\theta}_t = \hat{X}_t \tag{12}$$

where Z_t is obtained by optimally combining the Durbin (1954) and Pal (1980) estimators using GLS. The result is based on the Bayesian approach of Theil and Goldberger (1961). This leads to estimators that are more asymptotically efficient or at least as asymptotically efficient as using either only the Durbin or Pal estimators. This approach for obtaining Z_t is implemented in (8) above in deviation form.

Fama and MacBeth (1973) Two-Pass Method

The following discusses a typical test of the CAPM theory.

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[5] HAC is the heteroscedasticity and autocorrelation consistent estimator. We used the "Iterate to Convergence" Newey-West (1987) methodology of EViews 8.1.

[6] The assumption of a normally distributed matrix of errors is used to simplify the mathematical proof of the consistency of the estimators in this paper. This assumption is in no way a limitation in the modeling process of the time series used in this paper. Our proposed GMM_d estimator is based on the higher moments of the observed financial data and is thus able to capture the data's non-linearity, which is one of the important goals of this estimator.

One version of the Fama and MacBeth (1973)[7] two-pass method that practitioners and academics use is to test the CAPM based on three steps. (1) A surrogate for the market portfolio must be identified as it is not possible to find the theoretical market portfolio[8]. (2) For each asset, determine the beta via a first-pass regression. (3) Regress the mean returns of each asset on their betas. This represents the second-pass regression.

If the CAPM holds, then the second-pass regression should be the SML.

In this paper, we are doing only steps 1 and 2, as we are not directly testing the CAPM theory. One of our objectives is to investigate the dynamic properties of the alpha and beta across the business cycle while tackling specification error issues in the PS model as compared with the market model and the basic FF model. While we do believe that the Fama and MacBeth (1973) two-pass approach has significant merit, this issue in our view is a matter for another study.

Data and Descriptive Statistics

Data

Our sample is composed of monthly returns of 12 indices classified by FF industrial sectors. The observation periods are from July 1926 to December 2013 for a total 1,050 observations. The FF risk factors are drawn from French's website[9]. The PS tradable liquidity factor is from Pástor's website. The monthly data range from January 1968 to December 2013 for a total 552 observations[10].

Descriptive Statistics

Table 1 presents the descriptive statistics of the return variables for the 12 Fama-French sectors from July 1926 through December 2013.

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[7] See Benninga (2014), p. 276.
[8] See Roll (1977) for a discussion of the problems in testing the CAPM theory.
[9] French's website is http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[10] Pástor's website is http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2013.txt

Insert Table 1 here

The Jarque-Bera (1980) statistic is calculated by (13),

$$JB = (n-k)\left(\frac{skew^2}{6} + \frac{(kurt-3)^2}{24}\right) \sim \chi^2(2) \tag{13}$$

where n is the number of observations, k is the number of regressors which is zero when using the raw data, *skew* is the skewness of the data which is zero for a normal distribution, and *kurt* is the kurtosis which is three for the normal distribution. For all sectors, note that the JB statistic is greater than 5.99, which is the critical value of the chi-square distribution at the 5% level for 2 degrees of freedom. Thus, we reject the null hypothesis of normality for all sector returns. This is consistent with Mandelbrot (1963, 1972) and Fama (1963, 1965).

Sector 2 Durables has the highest standard deviation of 7.80, which would indicate that it is the riskiest sector on a standalone basis in the Markowitz (1959)[11] mean-variance framework. In the Rubinstein (1973) and Jurczenko and Maillet (2006) higher-moments framework, we note that this sector also has the highest kurtosis, which reinforces the idea that this sector may be the most risky. This is consistent with many consumers deferring durable purchases when the economy is weak and buying durables when the economy is strong. In other words, durable purchases are strongly cyclical.

Nine of the 12 sectors show positive skewness. Only 3 sectors show negative skewness, Sector 1 Non-durables -0.05, Sector 7 Telecommunications -0.02, and Sector 9 Shops -0.03. Negative skewness is an indicator of downside risk. Note, however, that in all three cases the negative skewness is close to zero.

Insert Table 2 here

Table 2 presents the descriptive statistics of the independent variables. The JB statistics are even more indicative of non-normality. The variables SMB, HML, and UMD (MOM), have extremely high JB

[11] Markowitz (2012) noted that the mean-variance model still works well in the presence of moderate amounts of skewness and kurtosis.

statistics, indicating that extreme events occur far more frequently than with the normal distribution. This is a reflection of the kurtosis measuring over 18 for each of these 3 variables, which is over 6 times the kurtosis of a normal distribution. The kurtosis of 10.35 and the JB of 2,357.19 for the market risk premium both fall within the respective ranges of the kurtosis and JB statistics from Table 1 for the sector returns. As we previously noted for the sector returns, even these JB values are well above the critical value of 5.99 that allows us to reject the null hypothesis of normality. The kurtosis of 5.39 and the JB statistic of 144.21 for the Pástor-Stambaugh LIQ factor are the lowest respective values for any of the dependent or independent variables in Tables 1 and 2. Nevertheless, this JB value of 144.21 is also well above the critical value[12].

All of these results suggest the logic of our proposed methodology which uses higher moments (cumulants) as instruments for the GMM estimation process. Using OLS when such strong non-normality is present in both the dependent and explanatory variables, may lead to incorrect inferences.

Is the Tradable Liquidity Factor a Proper Risk Factor?

In an unpublished paper, Baba *et. al*. (1990), later refined and published by Engle and Kroner (1995), proposed a parsimonious multivariate GARCH model (often called BEKK after the 4 original authors) to calculate the dynamic conditional correlation (DCC)[13]. Using this BEKK model, Figure 1 illustrates the DCC between the pair of variables SMB and LIQ.

Insert Fig. 1 here

Our conjecture is that SMB could be a proxy for LIQ. We therefore expect periods of significant correlation between these risk factors. As shown in Fig. 1, there are several periods where the correlation is significant. For instance, approximately in the period 1973-1975 the correlation is about 0.6 and in

[12] The LIQ variable is really a measure of illiquidity, not liquidity, as correctly noted by Bodie et. al. (2015, p. 406).

[13] For an introduction to non-linear models including multivariate GARCH processes with financial applications, see Racicot (2012).

2003 and 2012 it is above 0.5. Therefore, the correlation might be low on average, but it is useful in our view to look at it dynamically[14].

This casual observation is yet another motivation to examine rolling regressions with our new robust instruments incorporated into the generalized method of moments (GMM_d) versus ordinary least squares (OLS). The empirical results using these two approaches are discussed in the next section.

Empirical Results

Insert Fig. 2 here

The GMM_d procedure's sensitivity to disruptive influences is illustrated in Fig. 2, which use the average excess return values of the 12 FF sectors as the dependent variable. This figure compares the rolling alpha and beta, respectively, in the market model for both the OLS and GMM_d methods. For both OLS parameter estimates, the financial crisis of 2007-2009 had only minor impact. For both GMM_d parameter estimates, we observe that this financial crisis had substantial impact. These results are not surprising as the GMM_d method incorporates instruments that are based on higher moments of the independent variables.

Insert Fig. 3 here

Turning to Fig. 3 reveals that GMM_d is more sensitive to disruptive events for both the alpha and market beta for the rolling 60-month average based on using the average excess return values of the 12 FF sectors as the dependent variable in the PS model. The GMM_d alpha becomes extremely variable during the 1980-1986 great stagflation period of low economic growth and high inflation. It becomes noisy once again for an extended period that starts before the 2007-2009 financial crisis and ends after it. The GMM_d beta in Fig. 3 shows similar volatility for the 1980-1986 great stagflation period and for an extended period that spans the 2007-2009 financial crisis. Thus, extending the market model to the five-factor PS model, still leads us to conclude GMM_d estimates better capture disruptive events than the OLS

[14] We conducted other experiments where the DCC between the average of the returns of the FF 12 sector and SMB and the average and LIQ is computed. We find that the DCC correlation between the average and SMB is much higher than the one obtain for LIQ. This is further evidence that SMB could be a good proxy for LIQ.

estimates[15]. Furthermore, the increase in alpha and beta volatility in the PS model seems to anticipate the 2007-2009 financial crises

Tables 3 and 4 present the 60-month rolling estimated alpha and beta coefficients for the PS model using GMM_d and OLS, respectively, for the time period January 1968 through December 2013 for the FF telecommunications and money sectors as well as for the average of the 12 FF sectors. The telecommunications and money sectors have the lowest and highest average betas, respectively, of the 12 FF sectors. The data starts much later than for the market model because of the availability of the tradable LIQ data. Although only telecommunications and money are shown, the average of the rolling market beta coefficient

Insert Tables 3 and 4 here

is significant at 1% level for each of the 12 sectors using the OLS estimation method. For telecommunications, as well as for the not shown durables, energy, utilities, and health sectors, the average of the rolling market beta coefficient is significant at the 10% level using GMM_d. For the not shown non-durables, chemicals, business equipment, and shops, the average of the rolling market beta coefficient is significant at the 5% level. Only for money and the not shown manufacturing and other sectors is the average of the rolling market beta coefficient significant at the 1% level. Again, this seems to indicate that the GMM_d procedure is more sensitive to disruptive influences such as the subprime mortgage crisis of 2007-2009. The beta coefficients are not significantly different from zero for the SMB, HML, UMD (MOM), and LIQ factors for all 12 FF sectors for both the GMM_d and OLS estimation procedures.

Fig. 4 illustrates the variability in terms of adjusted R squared of the 60-month rolling GMM_d estimates for the

Insert Fig. 4

[15] Although not shown, similar results were obtained for the FF model.

market model compared to these estimates for the FF and PS models. Note that the adjusted R squared temporarily vanishes for the market model during the 2008-2009 financial crisis but declines only slightly for the FF and PS models. This suggests that at least during this crisis, some of the factors in these models have some explanatory power. Previously, we stated that the tradable LIQ factor was not significant on average. However, in a time of crisis, illiquidity usually rises substantially and may be a useful in explaining returns[16].

Conclusions

In this study, the dynamic properties of the CAPM alpha and beta are analyzed in the context of the Fama-French and Pástor-Stambaugh models using our new dynamic robust instruments GMM based estimator, including comparison to the rolling OLS estimator. This allowed us to examine how these dynamic estimators and associated inferences are related to phases of the business cycle, important issues that were not tackled in the static framework previously analyzed in the literature (e.g., Racicot and Rentz, 2015).

Principal conclusions are the following:

(1) The 60-month rolling market factor beta was more volatile for GMM_d than OLS for the market, Fama-French, and Pástor-Stambaugh models. This is shown most starkly in for the market model where the GMM_d beta is drastically affected by the 2007-2009 financial crisis. We believe using our new dynamic GMM_d approach based on higher moments of the variables is akin to the observed data since they show a high level of nonlinearity and non-spherical disturbances. Therefore, relying as previously done in the literature on OLS may result in incorrect estimations and related inferences.

(2) One conjecture is that liquidity (LIQ) is less potent in explaining the average sector returns than the small firm anomaly (SMB) and that SMB could be a replacement for LIQ. Our multivariate

[16] We have conducted further experiments using OLS to benchmark our results for the dynamic GMM_d. The results are quite similar although GMM_d is more sensitive to the financial crises observed in our sample, which is the virtues the dynamic estimator proposed in this article.

GARCH analysis suggests that most of the LIQ contribution to explaining excess return may already be priced in the SMB risk factor and that adding LIQ might create instability in the beta estimate for SMB.

Although we show in this paper that the LIQ variable may not have much econometric validity, this variable appears to improve the dynamic stability of the adjusted R^2 measure during periods of crisis. Furthermore, it appears that the GMM_d approach in the PS model has some ability to forecast the ensuing financial crisis of 2007-2009. This suggests that further research is warranted on the forecasting power of our approach.

This study is limited to the rolling regression approach and associated inferences of estimating the parameters across the phases of the business cycle. Other dynamic approaches are the recursive one and Kalman filtering. Further research should be done to investigate whether these may improve the efficiency and reliability of results.

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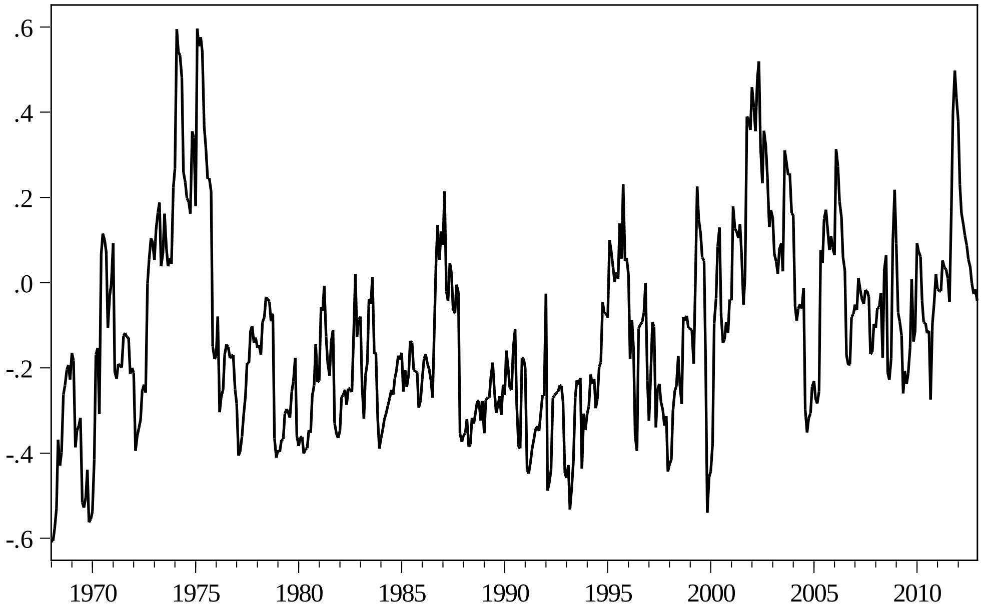

Fig. 1 Dynamic conditional correlation (DCC) between SMB and LIQ using BEKK-MGARCH
Source: Own computations in EViews 8.1 based on the multivariate GARCH model of Baba, Engle, Kraft, and Kroner (1990). French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html is the source for the SMB data from January 1968 through December 2013, and Pástor's website http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2013.txt is the LIQ source.

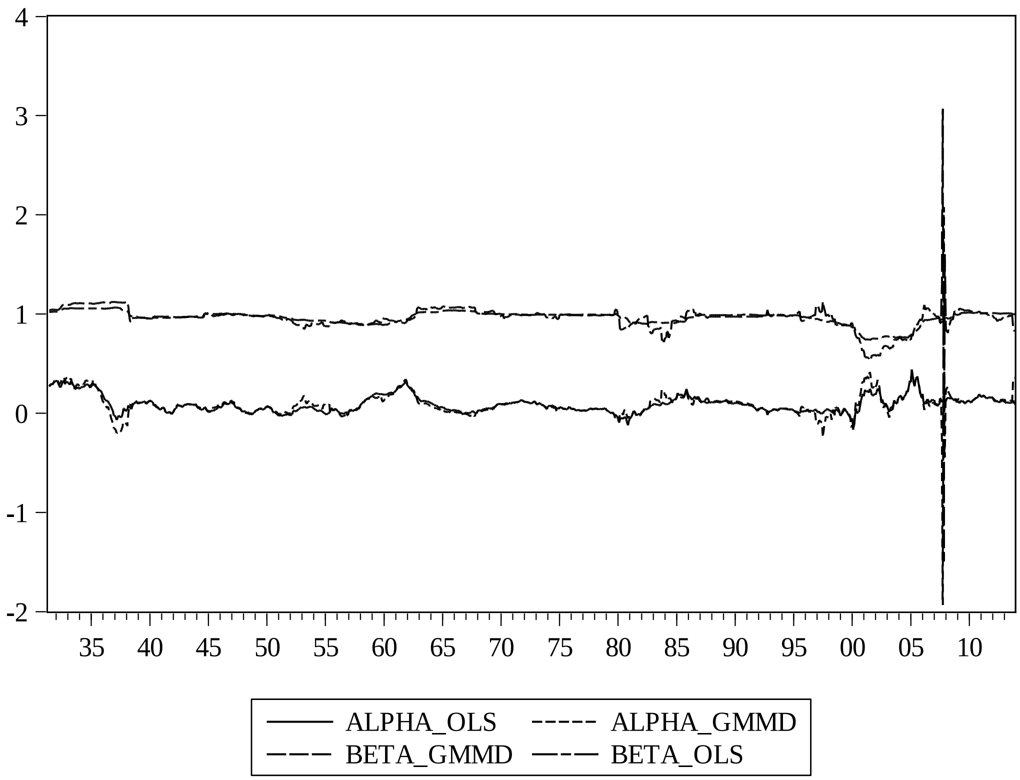

Fig. 2 Alpha and Beta GMM$_d$ and OLS estimations for the market model

Source: Own computations in EViews 8.1 for the average of the Fama-French 12 sectors returns with the sector and market returns from July 1926 through December 2013 obtained from French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

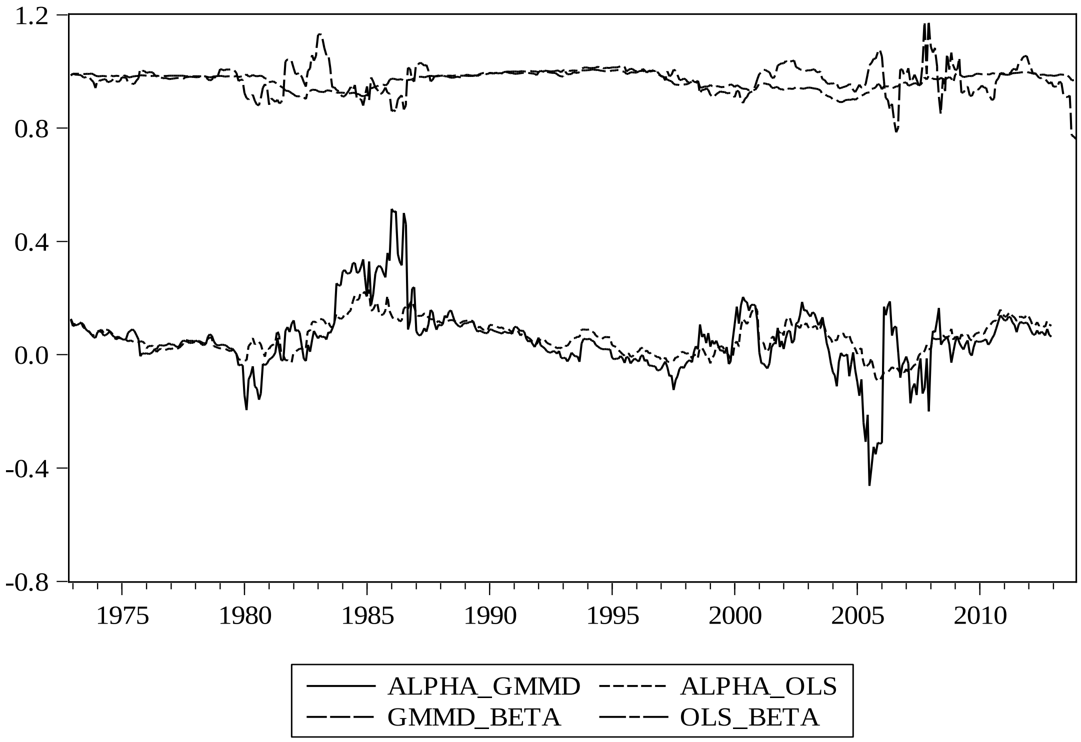

Fig. 3 Alpha and Beta GMM_d and OLS estimations for the PS model

Source: Own computations in EViews 8.1 for the average of the Fama-French 12 sectors returns from January 1968 through December 2013. Data for all variables were obtained from French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, except for tradeable LIQ which were obtained from Pástor's website http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2013.txt.

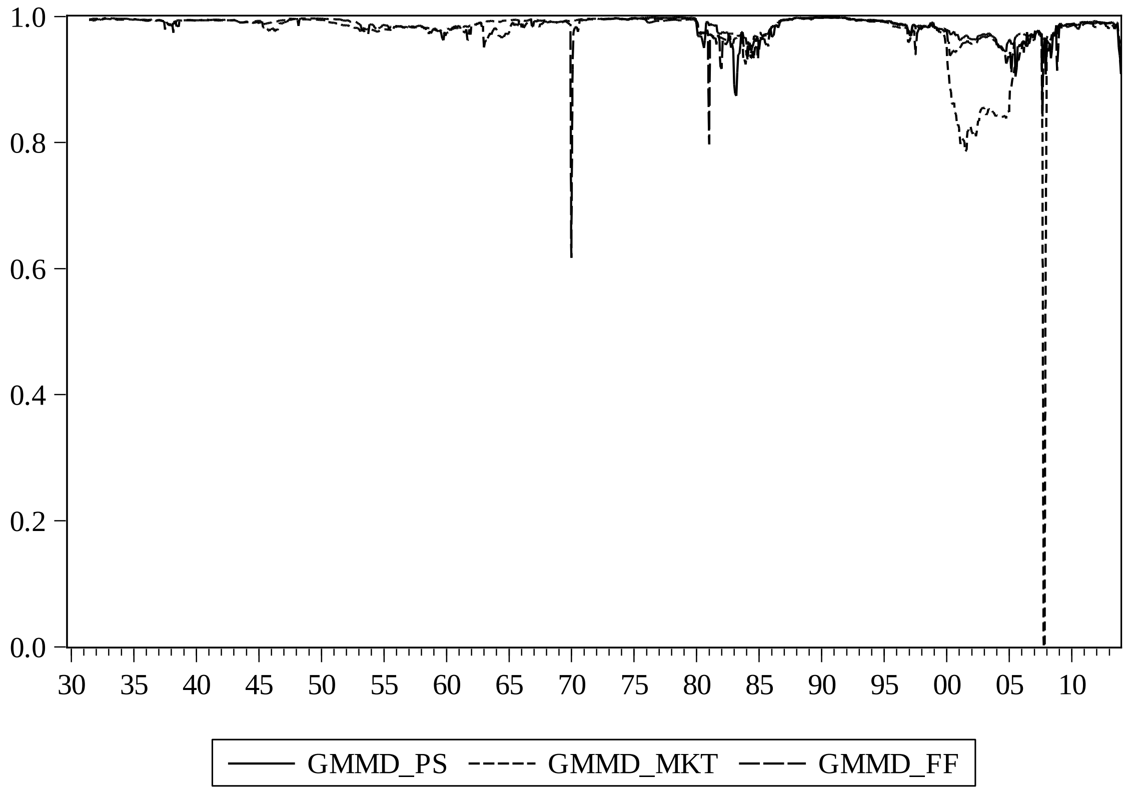

Fig. 4 Goodness of fit of the market, FF and PS models GMM_d adjusted R-squared
Source: Own computations in EViews 8.1 for the average of the Fama-French 12 sectors returns. Data for all variables from July 1926 through December 2013 were obtained from French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html, except for tradeable LIQ which were obtained from Pástor's website http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2013.txt from January 1968 through December 2013.

Table 1 Descriptive statistics for the Fama-French 12 sector factors 1926m07 – 2013m12

	Mean	Median	Max	Min	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
1 Nodur	0.99	1.12	34.39	-24.61	4.65	-0.05	8.77	1,458.77
2 Durbl	1.11	1.05	79.87	-34.82	7.80	1.12	16.96	8,742.62
3 Manuf	1.05	1.51	60.15	-28.83	6.78	0.95	15.23	6,699.94
4 Enrgy	1.06	0.91	33.47	-26.00	6.00	0.19	6.03	408.29
5 Chems	1.03	1.21	48.85	-31.62	5.80	0.38	11.47	3,161.20
6 Buseq	1.11	1.16	58.68	-34.63	7.60	0.43	10.12	2,251.54
7 Telcm	0.87	0.94	28.19	-21.56	4.63	-0.02	6.05	407.55
8 Utils	0.88	1.07	42.85	-32.85	5.58	0.06	10.63	2,549.52
9 Shops	1.02	1.15	42.25	-30.22	5.90	-0.03	9.06	1,607.60
10 Hlth	1.09	1.10	37.13	-34.08	5.65	0.10	9.59	1,903.02
11 Money	1.02	1.21	59.78	-39.59	6.88	0.51	14.18	5,516.67
12 Other	0.85	1.04	58.56	-31.22	6.64	0.85	15.35	6,801.53

Source: Own computations using EViews 8.1 based on 1,050 observations obtained from French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 2 Descriptive statistics for the Fama-French factors (1926m07–2013m12), Carhart momentum factor (1927m01–2013m12), and for the Pástor-Stambaugh tradable liquidity factor (1968m01–2013m12)

	Mean	Median	Max	Min	Std	Skewness	Kurtosis	Jarque-Bera
R_m-R_f	0.64	1.03	38.04	-29.10	5.43	0.16	10.35	2,357.19
SMB	0.24	0.08	37.45	-16.39	3.24	2.05	23.46	18,941.47
HML	0.39	0.24	34.08	-12.68	3.52	1.92	18.69	11,352.19
UMD	0.69	0.85	18.39	-52.15	4.77	-3.11	31.14	36,141.56
LIQ	0.00	0.00	0.21	-0.10	0.04	0.42	5.39	144.21

Source: Own computations using EViews 8.1 based on 552 observations for the tradable liquidity (LIQ) factor obtained from Pástor's website http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2013.txt and 1,044 observations for the remaining factors obtained from French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 3 Descriptive Statistics for Rolling Regression Parameter Estimations using GMM_d for the Pástor-Stambaugh Model 1968m01-2013m12 for the Telecom, Money, and 12-Sector Average

	GMM_d	Mean	Median	Max	Min	Std	Skewness	Kurtosis	JB-test
7 Telcm	α	0.08	0.02	2.68	-1.05	0.59	1.03	5.06	174.98
	β_{MKT}	0.79*	0.79	2.15	0.02	0.31	0.29	3.46	11.22
	β_{SMB}	-0.10	-0.18	1.00	-1.10	0.38	0.43	3.03	15.50
	β_{HML}	0.21	0.29	1.74	-1.62	0.46	-0.34	4.37	48.31
	β_{UMD}	0.00	-0.05	1.45	-0.93	0.35	0.92	5.38	185.40
	β_{LIQ}	3.93	-1.16	146.33	-124.11	43.71	0.26	3.05	5.65
11 Money	α	0.01	0.03	1.13	-1.02	0.43	0.06	2.41	7.34
	β_{MKT}	1.13***	1.13	1.85	-0.04	0.24	-0.60	4.86	101.24
	β_{SMB}	-0.13	-0.12	0.78	-0.89	0.27	-0.02	4.18	28.40
	β_{HML}	0.35	0.30	1.30	-0.54	0.33	0.26	2.77	6.53
	β_{UMD}	-0.14	-0.07	0.32	-0.80	0.24	-0.59	2.60	32.06
	β_{LIQ}	3.46	-7.63	106.03	-58.15	33.39	0.75	3.08	46.93
Avg	α	0.06	0.05	0.51	-0.46	0.11	0.16	7.10	347.64
	β_{MKT}	0.97***	0.98	1.18	0.76	0.05	-0.44	6.10	212.47
	β_{SMB}	0.02	0.00	0.65	-0.19	0.08	2.36	14.58	3,210.70
	β_{HML}	0.09	0.06	0.49	-0.19	0.15	1.12	3.71	114.01
	β_{UMD}	-0.05	-0.04	0.13	-0.29	0.06	-1.09	4.52	144.97
	β_{LIQ}	0.01	0.01	0.40	-0.18	0.08	1.89	9.73	1,222.03

Source: Own computations using EViews 8.1 based on 552 observations for the tradable liquidity factor (LIQ) obtained from Pastor's website http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2013.txt and 552 observations for the remaining factors obtained from French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Table 4 Descriptive Statistics for Rolling Regression Parameter Estimations using OLS for the Pástor-Stambaugh Model 1968m01-2013m12 for the Telecom, Money, and 12-Sector Average

	OLS	Mean	Median	Max	Min	Sd	Skewness	Kurtosis	JB-test
7 Telcm	α	0.06	0.08	0.85	-0.76	0.35	-0.12	2.36	9.65
	β_{MKT}	0.83***	0.85	1.38	0.33	0.20	-0.09	3.20	1.51
	β_{SMB}	-0.21	-0.25	0.25	-0.81	0.23	0.03	2.23	12.19
	β_{HML}	0.13	0.18	0.58	-0.48	0.25	-0.52	2.32	31.47
	β_{UMD}	-0.02	-0.05	0.38	-0.33	0.17	0.42	2.46	20.33
	β_{LIQ}	0.16	0.25	30.38	-29.08	10.20	-0.22	3.49	8.74
11 Money	α	-0.10	-0.09	0.56	-0.83	0.31	-0.22	2.23	16.10
	β_{MKT}	1.15***	1.13	1.45	0.87	0.12	0.46	2.98	17.09
	β_{SMB}	-0.02	-0.03	0.32	-0.36	0.16	0.11	2.16	15.47
	β_{HML}	0.34*	0.33	0.83	-0.09	0.20	0.31	2.44	14.53
	β_{UMD}	-0.09	-0.10	0.22	-0.44	0.14	-0.15	2.72	3.43
	β_{LIQ}	-2.79	-2.33	25.09	-30.93	13.82	-0.17	1.82	30.91
Avg	α	0.06	0.06	0.23	-0.09	0.06	0.01	2.79	0.94
	β_{MKT}	0.97***	0.98	1.01	0.89	0.03	-0.80	2.61	55.49
	β_{SMB}	0.01	0.00	0.15	-0.06	0.05	1.32	4.27	176.08
	β_{HML}	0.07	0.03	0.33	-0.09	0.11	1.27	3.51	137.51
	β_{UMD}	-0.04	-0.03	0.04	-0.18	0.04	-1.12	4.52	150.41
	β_{LIQ}	0.01	0.01	0.08	-0.05	0.03	0.37	2.28	22.15

Source: Own computations using EViews 8.1 based on 552 observations for the tradable liquidity factor (LIQ) obtained from Pastor's website http://faculty.chicagobooth.edu/lubos.pastor/research/liq_data_1962_2013.txt and 552 observations for the remaining factors obtained from French's website http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.