

## Préambule

La gestion financière responsable vise la maximisation de la richesse relative au risque dans le respect du bien commun des diverses parties prenantes, actuelles et futures, tant de l'entreprise que de l'économie en général. Bien que ce concept ne soit pas en contradiction avec la définition de la théorie financière moderne, les applications qui en découlent exigent un comportement à la fois financièrement et socialement responsable. La gestion responsable des risques financiers, le cadre réglementaire et les mécanismes de saine gouvernance doivent pallier aux lacunes d'un système parfois trop permissif et naïf à l'égard des actions des intervenants de la libre entreprise.

Or, certaines pratiques de l'industrie de la finance et de dirigeants d'entreprises ont été sévèrement critiquées depuis le début des années 2000. De la bulle technologique (2000) jusqu'à la mise en lumière de crimes financiers [Enron (2001) et Worldcom (2002)], en passant par la mauvaise évaluation des titres toxiques lors de la crise des subprimes (2007), la fragilité du secteur financier américain (2008) et le lourd endettement de certains pays souverains, la dernière décennie a été marquée par plusieurs événements qui font ressortir plusieurs éléments inadéquats de la gestion financière. Une gestion de risque plus responsable, une meilleure compréhension des comportements des gestionnaires, des modèles d'évaluation plus performants et complets intégrant des critères extra-financiers, l'établissement d'un cadre réglementaire axé sur la pérennité du bien commun d'une société constituent autant de pistes de solution auxquels doivent s'intéresser tant les académiciens que les professionnels de l'industrie. C'est en mettant à contribution tant le savoir scientifique et pratique que nous pourrons faire passer la finance responsable d'un positionnement en périphérie de la finance fondamentale à une place plus centrale. Le développement des connaissances en finance responsable est au cœur de la mission et des intérêts de recherche des membres du Groupe de Recherche en Finance Appliquée (GReFA) de l'Université de Sherbrooke.

Depuis la dernière crise financière de 2007-2009, le Comité de Bâle, dont plusieurs organismes de régulation financière sont membre, requiert des institutions financières qu'elles effectuent un monitoring plus serré de leurs risques financiers. De plus, les gestionnaires de fonds de placement tels que les fonds de pension sont également soumis à une gestion plus responsable des fonds qui leur sont confiés. Cette gestion requiert un suivi du risque encouru lors d'investissements ayant pour but de générer un rendement excédentaire afin de satisfaire les besoins de leurs clients. Dans cet article, nous proposons une nouvelle approche économétrique basée sur la méthode des moments généralisés (GMM) permettant de corriger le biais d'estimation occasionné, par exemple, par la présence d'erreurs de mesure ou plus généralement, d'erreurs de spécification causées par les problèmes d'endogénéités. Nous appliquons notre approche au nouveau modèle de Fama et French (2015) afin d'estimer, entre autres, la performance et le risque de portefeuilles de titres. De plus, nous y ajoutons le facteur de Pastor-Stambaugh (2003) afin de mesurer le risque d'illiquidité. Nous espérons que notre nouvelle approche permettra d'améliorer l'évaluation du risque et de la performance des fonds confiés aux gestionnaires de portefeuilles.

A panel data robust instrumental variable approach:  
a test of the new Fama-French five-factor model

April, 2016

François-Eric Racicot<sup>a,b,c</sup> William F. Rentz<sup>a</sup>

<sup>a</sup> *Telfer School of Management, University of Ottawa, Ottawa, ON K1N 6N5, Canada*

<sup>b</sup> *Groupe de Recherche en Finance Appliquée (GReFA), University of Sherbrooke, Canada*

<sup>c</sup> *CPA-Canada Accounting and Governance Research Centre, U. Ottawa; Corporate Reporting Chair, EGS-UQAM*

E-mail: [Racicot@telfer.uottawa.ca](mailto:Racicot@telfer.uottawa.ca)

## Abstract

Fama and French (FF, 2015) propose a new five-factor asset pricing model that adds profitability and investment patterns to the market, size, and value variables used in FF (1992). Our purpose is to investigate this new model using an improved GMM-based robust instrumental variables technique in a fixed effects panel data framework. To test for measurement errors, we use a modified Hausman artificial regression. We also examine an augmented FF six-factor model that includes the Pástor-Stambaugh (PS, 2003) liquidity factor. Using the FF data set, our GMM-based panel data approach leads us to conclude that the only consistently significant factor is the market factor.

**Keywords:** Fama-French five-factor model; GMM; fixed effects model; Pástor-Stambaugh liquidity; robust instruments

## 1. Introduction

Fama and French (FF, 1992) extended the capital asset pricing model of Sharpe (1964) to a three-factor asset pricing model that captured size and value in addition to the contribution of the excess market return. Recently, FF (2015) further extended their model to include two additional factors, profitability and investment. All five of these factors are represented by portfolios. These portfolio factors are presumed to span the space of the unknown state factors. This raises the

possibility of specification errors in the FF five-factor model. In particular, these factors may not span the space of the unknown state factors.

In addition to potential specification error, FF (2015, p. 2) state that the book/market ratio “is a noisy proxy for expected return”. Cochrane’s (2011, p. 1074) Q theory “links asset prices and investment”. Hou, Xue, and Zhan (2015) showed that Cochrane’s link can be modified to express a relation between expected returns and investment. Cochrane’s Q is approximated by the market/book ratio. This suggests that the FF value and investment factors are highly related.

Many factors in addition to FF’s five factors have been proposed in the literature. Perhaps the one that has received the most attention is the liquidity factor of Pástor-Stambaugh (PS, 2003). For this reason, we wish to compare the new FF model to an augmented six-factor model that includes the PS liquidity factor. The liquidity factor of Pástor-Stambaugh is a constructed variable that is a parameter obtained from a regression relating stock return to its trading volume. Pagan (1984, 1986) shows that constructed variables may increase the variance of the OLS estimator but the estimator remains unbiased.

The generalized method of moments (GMM) developed by Hansen (1982) provides a potent solution to the problems of specification and measurement errors. However, the problem of weak instruments has more recently put in doubt the applicability of this method. When instrumental variables are weak, the two-stage least squares (2SLS) estimator is inconsistent (Nelson and Starz, 1990a,b; Bound, Jaeger, and Baker, 1995; Hahn and Hausman, 2003).

Several researchers (e.g. Dagenais and Dagenais, 1997; Racicot and Théoret, 2014) responded to the problem of weak instruments by developing a method that generates instruments that show greater robustness. These instruments are computed using a Bayesian averaging process (Theil and Goldberger, 1961) of two generalized versions of Durbin (1954) and Pal (1980) higher moment estimators. Two features of this approach are its i) is a parsimonious because it does not require much computational power and ii) can be viewed as minimizing a distance ( $d$ ) measure. Hence, we refer to this GMM approach as  $GMM_d$ .

In this paper, we develop an empirical extension to our previous theoretical model (Racicot, 2015) that validates the  $GMM_d$  approach in a fixed effects panel data framework. This extension allows us to i) study the robustness of the new FF (2015) model and ii) compare this model to an augmented model using the PS (2003) liquidity factor. With this empirical work, we are able to shed some light on the problem of unobserved heterogeneity in panel data models that

may exacerbate measurement errors when not treated properly. The method of first-differencing to remove unobserved heterogeneity may actually worsen the situation. In fact, it is only by chance that first-differencing in a panel data framework will attenuate measurement errors (Arrenallo, 2003).

The rest of this article is constructed as follows. Section 2 provides a review of the basic fixed effects panel data framework in the context of errors in variables for the new game-French (FF, 2015) five-factor model and the augmented FF six-factor model that includes Pástor-Stambaugh (PS, 2003) liquidity. Section 3 discusses the GMM<sub>d</sub> approach for the panel data framework. Section 4 presents our empirical results. Section 5 presents our conclusions and suggestions for further research.

## 2. Fixed Effects Framework for Fama-French Five-Factor and Augmented Six-Factor Models

### *Five- and six-factor models<sup>1</sup>*

Fama and French (2015) introduced the following five-factor model<sup>2</sup>.

$$R_{it} - R_{Ft} = a_i + b_i (R_{Mt} - R_{Ft}) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + e_{it} \quad (1)$$

The first three factors  $R_{it} - R_{Ft}$ ,  $SMB_t$ , and  $HML_t$  are the well-known market, size, and value factors introduced in FF (1992). The factor  $RMW_t$  is the difference in returns in period  $t$  of diversified portfolios of stocks with robust and weak profitability. The factor  $CMA_t$  is the difference in returns in period  $t$  of diversified portfolios of conservative and aggressive firms with respect to investment behavior.

The  $HML_t$  factor represents the difference in returns in period  $t$  of diversified portfolios of stocks with high book-to-market ratios and low book-to-market ratios. (2) below illustrates why book-to-market or B/M ratios are related to the rate of return of a financial asset.

$$\frac{M_t}{B_t} = \frac{\sum_{\tau=1}^{\infty} E(NI_{t+\tau} - \Delta B_{t+\tau}) / (1+r)^\tau}{B_t} \quad (2)$$

$E(\cdot)$  is the expectation operator,  $NI_{t+\tau}$  is the net income for period  $t + \tau$ ,  $\Delta B_{t+\tau} = B_{t+\tau} - B_{t+\tau-1}$  is the change in total book value of equity, and  $r$  is the return on the financial asset. (2) may also be

<sup>1</sup> Here we follow our previously developed approach (Racicot and Rentz, 2016).

<sup>2</sup> The data for the five FF factors and the market and sector returns are available from [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

viewed as a proxy for Tobin's (1969) Q, which is the market value of installed capital divided by its replacement cost.

In the same vein, a firm invests more when its marginal Q is high<sup>3</sup>. Intuitively, as a firm invests more, it will move down its investment opportunity schedule until the marginal benefit equals the marginal cost. Thus, higher investment for a firm drives down its rate of return. This leads to the following equation (Hou *et. al.*, 2015) for stock  $i$  at time  $t$ ,

$$E_t(r_{it}) = \frac{E_t(\Pi_{it+1})}{1 + a(I_{it} / A_{it})} \quad (3)$$

where  $E_t(r_{it})$  and  $E_t(\Pi_{it+1})$  are the conditional expected return and profitability, respectively;  $a$  is a parameter for adjustment costs;  $I_{it}$  is the investment; and  $A_{it}$  are the firm's productive assets. This model is based on a stochastic general equilibrium model in a two-period setting (Lin and Zhang, 2013), where the rate of return on investment is equated to the firm's discount rate or cost of capital. (3) may be viewed as rationale for the factors  $RMW_t$  and  $CMA_t$  in (1).

Pástor and Stambaugh (2003) introduced a liquidity factor  $LIQ_t$  to the original Fama and French (1992) three-factor model. The Pástor-Stambaugh liquidity factor is a constructed variable.  $LIQ_t$  is an average of the stock  $\hat{\gamma}_{it}$  obtained from regression (4).

$$r_{id+1t} - r_{md+1t} = \theta_{it} + \varphi_{it}r_{idt} + \gamma_{it}sign(r_{idt} - r_{mdt})v_{idt} + \varepsilon_{id+1t} \quad (4)$$

where  $r_{idt}$  is the return of stock  $i$  on day  $d$  in month  $t$  and  $v_{idt}$  is the dollar trading volume of stock  $i$  on day  $d$  in month  $t$ . Pagan (1984, 1986) shows that constructed variables may increase the variance of the OLS estimator but the estimator remains unbiased. In this paper, we compare (1) with an augmented version of this equation that includes the liquidity as a sixth factor<sup>4</sup>.

We extend the model in (1) to a fixed effects panel data framework including the LIQ factor in (5) below, written in a stacked vector format for the 12 FF sectors.

$$Y = \mathbf{R} - \mathbf{R}_F = \sum_{i=1}^{12} \alpha_i \mathbf{D}_i + \sum_{i=1}^{12} \beta_i \mathbf{D}_i (\mathbf{R}_M - \mathbf{R}_F) + s \mathbf{SMB} + h \mathbf{HML} + r \mathbf{RMW} + c \mathbf{CMA} + l \mathbf{LIQ} + e \quad (5)$$

<sup>3</sup> The marginal Q is the NPV of future cash flows generated from an additional unit of assets. Note that (3) is derived equating the marginal benefit to marginal cost.

<sup>4</sup> The LIQ factor is available from Pastor's website <http://faculty.chicagobooth.edu/lubos.pastor/research/>. We use the tradable LIQ factor and multiply it by 100 to put it in percentage form.

$Y' = (R_{11} - R_{F1}, \dots, R_{1T} - R_{FT}, \dots, \dots, R_{12,1} - R_{F1}, \dots, R_{12,T} - R_{FT})$  represents the transpose of the stacked vector  $Y$  of excess returns for each sector.  $D_i' = (0, \dots, 0, \dots, 1, \dots, 1, 0, \dots, 0)$  is the transpose of the stacked dummy variable, which is 0 everywhere except for the  $T$  observations for sector  $i$ .  $\alpha_i$  is the Jensen (1968) performance measure for sector  $i$ .

$(R_M - R_F)' = (R_{M1} - R_{F1}, \dots, R_{MT} - R_{FT}, \dots, R_{M1} - R_{F1}, \dots, R_{MT} - R_{FT})$  is the transpose of the stacked vector of excess market returns. That is, the excess market returns are stacked 12 times, once for each sector.  $\beta_i$  is the sector  $i$  CAPM systematic risk beta. The other explanatory variables are similarly defined. The coefficients of these other variables are 12-sector pooled coefficients.  $e$  is the stacked vector of error terms.

### 3. GMM<sub>d</sub> Approach and Hausa Test in a Panel Data Framework<sup>5</sup>

#### *Fixed effects model*

The GMM estimator for estimating the fixed effects panel data regression models is given by

$$\hat{\theta}_{GMM_d} = \left[ \left( \sum_{i=1}^N X_i' d_i \right) \hat{w}^{-1} \left( \sum_{i=1}^N d_i' X_i \right) \right]^{-1} \left[ \left( \sum_{i=1}^N X_i' d_i \right) \hat{w}^{-1} \left( \sum_{i=1}^N d_i' Y_i \right) \right] \quad (6)$$

where  $d_i = x_i - \hat{x}_i$  is a vector of robust “distance” instruments. We compute these instruments by applying generalized least-squares (GLS) to a combination of two robust estimators [Durbin (1954) and Pal (1980) estimators]. These estimators are respectively defined by  $\beta_D = (z_1' x)^{-1} (z_1' y)$  (Durbin),  $\beta_P = (z_2' x)^{-1} (z_2' y)$  (Pal), where  $z_1 = [x_{ikt}^2]$ ,  $z_2 = z_3 - 3 \text{Diag}(x' x / N) x'$ ,  $z_3 = [x_{ikt}^3]$ , and  $\text{Diag}(x' x / N) = x' x / N \bullet I_k$  are stacked vectors with  $i$  representing the sectors ( $i = 1, \dots, N$ ),  $k$  the number of explanatory variables (either 5 or 6), and  $t$  the time subscript ( $t = 1, \dots, T$ ). The notation  $\bullet$  is the Hadamard product. The second and third power (moments) of the *de-meaned* variables ( $x$ ) are then computed. Then the weighted estimator ( $\beta_H$ ) is obtained by an application of the GLS to the following combination  $\beta_H = W \begin{pmatrix} \beta_D \\ \beta_P \end{pmatrix}$  where  $W = (C' S^{-1} C)^{-1} C' S^{-1}$  is the GLS weighting matrix,  $S$  is the covariance matrix of  $\begin{pmatrix} \beta_D \\ \beta_P \end{pmatrix}$  under the null

<sup>5</sup> For more information, see Racicot (2015). Note that the dummy variable in (5) is excluded from (6) below.

hypothesis (i.e., no measurement errors), and  $\mathbf{C} = \begin{pmatrix} \mathbf{I}_k \\ \mathbf{I}_k \end{pmatrix}$  is a matrix of two stacked identity matrices of dimension  $k$ . This GLS approach is *optimal* in the Aitken (1935) sense<sup>6</sup>. However, we opt for the GMM method to weight the Durbin and Pal’s estimators.

To implement the GMM<sub>d</sub> approach in a fixed effects panel data framework, first create the dummy variables for each sector. Next compute the robust instruments using the above algorithm. Then calculate the GMM estimators using a HAC matrix with the newly computed robust instruments and the sector dummy instruments.

#### *Haus<sub>d</sub> test for measurement errors*

To test whether there are measurement errors, we rely on a modified Hausman (1978) artificial regression which we refer to as Haus<sub>d</sub>. Each variable in the original five-factor and six-factor models has a companion variable in Haus<sub>d</sub> with its own  $t$  statistic that indicates whether the original variable contains measurement error.

## **4. Empirical Results**

### *FF five-factor model*

For the fixed effects FF five-factor model, (see table) the Jensen (1968) measure  $\alpha$  of performance is significant for only 2 sectors (Business Equipment and Other) using OLS, for only 1 sector (Other) using GMM<sub>d</sub>, and for only 1 sector (Other) using Haus<sub>d</sub>. The average of the 12 values of  $\alpha$  is not significant for any of the methods. Note that the OLS average value of  $\alpha$  is identical to the  $\alpha$  obtained from the pooled regression model.

**Insert Table here**

This result generalizes to the beta matrix for all coefficients as shown in (7).

$$\boldsymbol{\beta}^{total} = \mathbf{F}^{within} \boldsymbol{\beta}^{within} + \mathbf{F}^{between} \boldsymbol{\beta}^{between} \quad (7)$$

This equation shows that the pooled estimate is an average of the within (same time period) and between (individual sectors) estimates<sup>7</sup>.

---

<sup>6</sup>Note that  $\mathbf{W}$  can be replaced by the White (1980) or the Newey-West (1987) HAC asymptotically consistent variance-covariance matrix. In this article, we use the HAC matrix.

<sup>7</sup> For a discussion of the calculation of  $\mathbf{F}$ , see Greene (2012), p. 358.

The measure of relative systematic risk  $\beta$  is significant at the 1% level or better for all 12 sectors in the FF model regardless of which estimation method is employed. The coefficient of the size factor (*SMB*) is significant at the 10% level using OLS. The coefficients of the three other FF factors, value (*HML*), profitability (*RMW*), and investment (*CMA*) are all significant at the 1% level using OLS. This appears to offer strong support for 4 of the 5 FF factors with more modest support for *SMB*.

Using GMM<sub>d</sub>, however, tells a different story for the 4 factors *SMB*, *HML*, *RMW*, and *CMA*. The *RMW* factor is significant at the 5% level, but none of the other three factors are significant.

Turning to Haus<sub>d</sub>, *SMB*, *HML*, and *CMA* remain insignificant. The coefficient of *RMW* improves to the 1% level. The variable  $\hat{\omega}_{RMW}$  is an instrument that is built upon the higher moments (a proxy of cross skewness and cross kurtosis) of the sample and is primarily related to *RMW*. Note that it is significant at the 5% level. This is an indication of measurement errors in the *RMW* factor and/or possible non-normality of the *RMW* factor. Recall that the rationale for *RMW* is based on expectations in (3). Thus, *RMW* is a proxy for the underlying unobservable expectations.

#### *FF six-factor model*

For the fixed effects FF five-factor model augmented by the PS (2003) liquidity factor, (see table) the Jensen measure  $\alpha$  of performance is once again significant for only the same 2 sectors (Business Equipment and Other) using OLS and for only the same single sector (Other) using GMM<sub>d</sub>, or Haus<sub>d</sub>. The average of the 12 values of  $\alpha$  is not significant for any of the methods.

Once again the measure of relative systematic risk  $\beta$  is significant at the 1% level or better for all 12 sectors in the FF model regardless of which estimation method is employed. *SMB* is again significant at the 10% level using OLS, and the three other FF factors, *HML*, *RMW*, and *CMA* are again all significant at the 1% level using OLS. The *LIQ* factor, however, is not significant. This appears to again offer strong support for 4 of the 5 FF factors with more modest support for *SMB*. The *LIQ* factor, however, seems to add little to the model, as the coefficients of the original variables are essentially the same and the adjusted  $R^2$  remains unchanged at 0.69.

Using GMM<sub>d</sub>, none of the variables are significant except for the previously discussed excess market return factor. However, when using Haus<sub>d</sub>, *LIQ* becomes significant but measured with errors as indicated by the significance of  $\hat{\omega}_{LIQ}$ . This should not be surprising, since *LIQ* is a



constructed variable as shown in (4). Note also that *CMA* is significant at the 10% level using Haus<sub>d</sub>.

## 5. Conclusions

This article uses an innovative fixed effects panel data approach for estimating the parameters of the new Fama and French (2015) model where measurement errors are suspected. For both the five-factor FF model and the six-factor augmented FF model that includes the Pástor-Stambaugh (2003) liquidity factor, the excess market return factor is significant at the 1% level for all 12 FF sectors regardless of whether OLS, GMM<sub>d</sub>, or Haus<sub>d</sub> is used.

While we have reported some evidence to support the *SMB*, *HML*, *RMW*, *CMA*, and *LIQ* factors, the significance of each of these factors is highly variable.

## References

- Aitken, A.C. (1935) On least squares and linear combinations of observations, *Proceedings of the Royal Statistical Society*, 55, 42-48.
- Arellano, M. (2003) Panel Data Econometrics. *Advanced Text in Econometrics*, Oxford University Press.
- Bound, J., Jaeger, D., Baker, R. (1995) Problems with instrumental variables estimation when the correlation between the instruments and the endogenous explanatory variables is weak, *Journal of the American Statistical Association*, 90, 443-450.
- Cochrane, J.H. (2011) Presidential address: Discount rates, *Journal of Finance*, 66(4), 1047-1108.
- Dagenais, M.G., Dagenais, D.L. (1997) Higher moments estimators for linear regression models with errors in the variables, *Journal of Econometrics*, 76(1-2), 193-221.
- Durbin, J. (1954) Errors in variables, *International Statistical Review*, 22(1/3), 23-32.
- Fama, E.F., French, K.R. (1992) The cross-section of expected stock returns, *Journal of Finance*, 47(2), 427-465.
- Fama, E.F., French, K.R. (2015) A five-factor asset pricing model, *Journal of Financial Economics*, 116, 1-22.
- Greene, W.H. (2012) *Econometric Analysis*, 7<sup>th</sup> ed., Pearson.
- Hahn, J., Hausman, J. (2003) Weak instruments: diagnosis and cures in empirical economics, *American Economic Review*, 93(2), 118-125.
- Hansen, L. (1982) Large sample properties of the generalized method of moments estimators, *Econometrica*, 50(4), 1029-1054.
- Hausman, J.A. (1978) Specification tests in econometrics, *Econometrica*, 46(6), 1251-1271.
- Hou, K., Xue, C., Zhang, L. (2015) Digesting anomalies: an investment approach, *Review of Financial Studies*, 28(3), 650-705.
- Jensen, M.C. (1968) The performance of mutual funds in the period 145-1964, *Journal of Finance*, 23(2), 389-416.
- Lin, X., Zhang, L. (2013) The investment manifesto, *Journal of Monetary Economics*, 60, 351-366.
- Nelson, C., Startz, R. (1990a) Some further results on the exact small sample properties of the instrumental variables estimator, *Econometrica*, 58(4), 967-976.

- Nelson, C., Startz, R. (1990b) The distribution of the instrumental variables estimator and its  $t$ -ratio with the instrument is a poor one, *Journal of Business*, 63(1), S125-S140.
- Newey, W., West, K. (1987) A simple positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica*, 55(3), 703-708.
- Pagan, A.R. (1984) Econometric issues in the analysis of regressions with generated regressors, *International Economic Review*, 25(1), 221-247.
- Pagan, A.R. (1986) Two stage and related estimators and their applications, *Review of Economic Studies*, 53(4), 517-538.
- Pal, M. (1980) Consistent moment estimators of regression coefficients in the presence of errors in variables, *Journal of Econometrics*, 14(3), 349-364.
- Pástor, L., Stambaugh, R.F. (2003) Liquidity risk and expected stock returns, *Journal of Political Economy*, 111(3), 643-685.
- Racicot, F.E. (2015) Engineering robust instruments for panel data regression regression models with errors in variables, *Applied Economics*, 47(10), 9-30.
- Racicot, F.E., Rentz, W.F. (2016) Testing Fama-French's new five-factor asset pricing model: evidence from robust instruments, *Applied Economics Letters*, 23(6): 444-448.
- Racicot, F.E., Théoret, R. (2014) Cumulant instrument estimators for hedge fund return models with errors in variables, *Applied Economics*, 46(10), 1134-1149.
- Sharpe, W.F. (1964) Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance*, 19, 425-442.
- Theil, H., Goldberger, A.S. (1961) On pure and mixed statistical estimation in economics, *International Economic Review*, 2(2), 65-78.
- Tobin, J. (1969) A general equilibrium approach to monetary theory, *Journal of Money, Credit and Banking*, 1(1), 15-29.
- White, H. (1980) A heteroscedasticity-consistent covariance matrix estimator and a direct test for heteroscedasticity, *Econometrica*, 48, 817-838.

Table: Comparing OLS and GMM<sub>d</sub> estimations of fixed effects model for the new Fama-French (2015) model and its illiquidity extended version

	<i>Alpha</i>	<i>R<sub>m</sub>-R<sub>f</sub></i>	<i>SMB</i>	<i>HML</i>	<i>RMW</i>	<i>CMA</i>	<i>LIQ</i>	$\hat{\omega}_{R_m-R_f}$	$\hat{\omega}_{SMB}$	$\hat{\omega}_{HML}$	$\hat{\omega}_{RMW}$	$\hat{\omega}_{CMA}$	$\hat{\omega}_{LIQ}$	$\bar{R}^2$	<i>DW</i>
<b>Fama-French 5 fac.</b>															
OLS	-0.0492*	0.9928*	0.0216	0.1091	0.1671	0.0748								0.69	1.94
<i>t-stat</i>	<i>1.02</i>	<i>35.82</i>	<i>1.67</i>	<i>6.09</i>	<i>8.97</i>	<i>2.69</i>									
Abs <i>t</i> -min	<i>0.02</i>	<i>20.72</i>													
Abs <i>t</i> -max	<i>2.61</i>	<i>47.19</i>													
# of signif. indices	2	12													
GMM <sub>d</sub>	-0.0614*	0.9374*	0.0906	-0.0016	0.3976	0.0901								0.67	1.93
<i>t-stat</i>	<i>1.03</i>	<i>7.67</i>	<i>0.97</i>	<i>-0.01</i>	<i>2.04</i>	<i>0.47</i>									
Abs <i>t</i> -min	<i>0.03</i>	<i>2.23</i>													
Abs <i>t</i> -max	<i>3.24</i>	<i>17.33</i>													
# of signif. indices	1	12													
Haus <sub>d</sub>	-0.0238*	0.9374*	0.0906	-0.0016	0.3976	0.0901	0.0542	-0.0670	0.0927	-0.2805	-0.0292			0.69	1.94
<i>t-stat</i>	<i>0.99</i>	<i>11.66</i>	<i>1.53</i>	<i>-0.02</i>	<i>3.40</i>	<i>0.84</i>	<i>1.24</i>	<i>-1.10</i>	<i>1.03</i>	<i>-2.36</i>	<i>-0.26</i>				
Abs <i>t</i> -min	<i>0.07</i>	<i>3.29</i>					<i>0.17</i>								
Abs <i>t</i> -max	<i>2.36</i>	<i>18.46</i>					<i>4.07</i>								
# of signif. indices	1	12					3								
<b>FF 5 fac. + LIQ</b>															
OLS	-0.0535*	0.9932*	0.0218	0.1090	0.1669	0.0748	0.0098							0.69	1.94
<i>t-stat</i>	<i>1.03</i>	<i>35.83</i>	<i>1.68</i>	<i>6.08</i>	<i>8.96</i>	<i>2.69</i>	<i>0.95</i>								
Abs <i>t</i> -min	<i>0.06</i>	<i>20.73</i>													
Abs <i>t</i> -max	<i>2.64</i>	<i>47.20</i>													
# of signif. indices	2	12													
GMM <sub>d</sub>	-0.1653*	1.0148*	-0.0501	0.1697	0.1345	0.2242	0.1152							0.65	1.89
<i>t-stat</i>	<i>1.99</i>	<i>7.77</i>	<i>-0.39</i>	<i>0.79</i>	<i>0.52</i>	<i>1.20</i>	<i>1.34</i>								
Abs <i>t</i> -min	<i>0.02</i>	<i>2.74</i>													
Abs <i>t</i> -max	<i>3.55</i>	<i>14.62</i>													
# of signif. indices	2	12													
Haus <sub>d</sub>	-0.0256*	1.0148*	-0.0501	0.1697	0.1345	0.2242	0.1152	-0.0232	0.0740	-0.0790	-0.0179	-0.1632	-0.1105	0.69	1.94
<i>t-stat</i>	<i>0.99</i>	<i>11.76</i>	<i>-0.61</i>	<i>1.52</i>	<i>0.85</i>	<i>1.86</i>	<i>2.46</i>	<i>1.16</i>	<i>0.89</i>	<i>-0.69</i>	<i>-0.11</i>	<i>-1.32</i>	<i>-2.30</i>		
Abs <i>t</i> -min	<i>0.09</i>	<i>3.96</i>						<i>0.20</i>							
Abs <i>t</i> -max	<i>2.37</i>	<i>18.09</i>						<i>3.18</i>							
# of signif. indices	1	12						1							

\*The average of the coefficients obtained from the fixed effects model is also the pooled value. The average of the *t*-statistics is computed from the absolute values.

Notes: The *t*-statistics are in italic. The *t*-statistics for the GMM<sub>d</sub> are computed using the Newey-West (1987) HAC matrix. # of signif. indices represents the number of significant FF sectors at the 5% level.  $\bar{R}^2$  is the adjusted *R*-squared and *DW* is the Durbin-Watson statistic. The companion variables in the Hausman artificial regression are represented by  $\hat{\omega}$ . When the *t*-statistic for a companion variable is significant at the 5% level, this suggests evidence that the original variable might be measured with error.